PURE MATHS 4 (A) TEST PAPER 6 : ANSWERS AND MARK SCHEME

1.	$\sum (r^3 + r) = n^2(n+1)^2/4 + n(n+1)/2 = n(n+1)(n^2+n+2)/4$	M1 A1 M1 A1	
	$a = 1/4,\ b = 2$	A1	5
2.	$\frac{w^2}{z} = \frac{(-1+5i)(-1+5i)(4+2i)}{(4-2i)(4+2i)} = \frac{(-24-10i)(4+2i)}{20} = -\frac{19}{5} - \frac{22}{5}i$	M1 A1 A1 M1 A1 A1	6
3.	Aux. eqn. is $3u^2 - 5u - 12 = 0$ $\quad (3u+4)(u-3) = 0$	B1 M1	
	$u = -4/3,\ u = 3$ $\quad$ General solution is $y = ae^{-4x/3} + be^{3x}$	A1 M1 A1 A1	6
4.	By graphs or otherwise, $x < -3,\ -1 < x < 0,\ x > 1$	M3 A1 A1 A1 A1	7
5.	(a) $f(1) = -1{\cdot}149,\ f(1{\cdot}4) = 1{\cdot}683$, so root in $(1, 1{\cdot}4)$	M1 A1	
	(b) $f'(x) = \sec x \tan x - 3$ $\quad 1{\cdot}3 - f(1{\cdot}3)/f'(1{\cdot}3) = 1{\cdot}313$	M1 A1 M1 A1	
	$1{\cdot}313 - f(1{\cdot}313)/f'(1{\cdot}313) = 1{\cdot}314$ (to 3 d.p.)	M1 A1	8
6.	(a) $\lvert z_1 \rvert = \lvert z_2 \rvert = 5\sqrt{2},\ \lvert z_3 \rvert = 5$	B1 B1 B1	
	$\arg(z_1) = 7\pi/4 = -0{\cdot}79,\ \arg(z_2) = 3{\cdot}00,\ \arg(z_3) = 0{\cdot}93$	B1 B1 B1	
	(b) Sides are given by vectors (10 3), (2 −9), (12 −6)	B1 B1 B1	
	Perimeter $= \sqrt{180} + \sqrt{85} + \sqrt{109} \approx 33{\cdot}1$	M1 A1	
	(c) Reflection in real axis (x -axis)	B1 B1	13
7.	(a) $y = r\sin\theta = 3\sin 2\theta - \sin\theta$ $\quad dy/d\theta = 6\cos 2\theta - \cos\theta = 0$	B1 M1 A1	
	$12\cos^2\theta - \cos\theta - 6 = 0$ $\quad (4\cos\theta - 3)(3\cos\theta + 2) = 0$	M1 A1	
	$\cos\theta = 3/4$ or $-2/3$ $\quad$ Then $r = 3{\cdot}5$ or $r = -5$ (accept 5)	M1 A1 (both)	
	(b) Area $= 2 \times \frac{1}{2}\int_0^a (36\cos^2\theta - 12\cos\theta + 1)\,d\theta$ where $a = \arccos(1/6)$	M1 A1 B1	
	$= \int_0^a (18\cos 2\theta - 12\cos\theta + 19)\,d\theta = [9\sin 2\theta - 12\sin\theta + 19\theta]_0^a$	M1 A1	
	$= 6\sin a(3\cos a - 2) + 19a = 19\arccos(1/6) - 3(\sqrt{35})/2 \approx 17{\cdot}8$	M1 A1 A1	15
8.	(a) With $v = y^4$, $\frac{dv}{dx} = 4y^3\frac{dy}{dx}$ $\quad$ Eqn. is $\frac{1}{4y^3}\frac{dv}{dx} - \frac{y}{2x} = \frac{5x^2}{y^3}$	B1 M1 A1	
	$\frac{dv}{dx} - \frac{2y^4}{x} = 20x^2$ $\quad \frac{dv}{dx} - \frac{2}{x}v = 20x^2$	M1 A1	
	(b) I.F. $= e^{-2\ln x} = x^{-2}$ $\quad \frac{1}{x^2}\frac{dv}{dx} - \frac{2}{x^3}v = 20$ $\quad \frac{d}{dx}\left(\frac{v}{x^2}\right) = 20$	B1 M1 A1	
	$\frac{v}{x^2} = 20x + c$ $\quad v = 20x^3 + cx^2$ $\quad y = (20x^3 + cx^2)^{1/4}$	M1 A1 A1	
	(c) $y(1) = 2$; $\quad 20 + c = 16$ $\quad c = -4$ $\quad$ Need $v > 0$, so $x > 1/5$	M1 A1 M1 A1	15