PURE MATHS 4 (A) TEST PAPER 7 : ANSWERS AND MARK SCHEME

1.	$-0{\cdot}8 + (0{\cdot}312)/(0{\cdot}687) \times 0{\cdot}3 = -0{\cdot}66$	M1 A1 M1 A1 A1	5
2.	$x = r\cos\theta = ae^{\theta}\cos\theta$ $\quad$ $dx/d\theta = ae^{\theta}(\cos\theta - \sin\theta) = 0$ at	B1 M1 A1 M1	
	required points $\quad$ $\cos\theta = \sin\theta$ $\quad$ $\tan\theta = 1$ $\quad$ $\theta = \pi/4$	A1 A1	6
3.	$1 - x^2 < 1 - x < x^2 - 1$ $\quad$ $x^2 - x > 0$ and $x^2 + x - 2 > 0$	B1 M1 A1 A1	
	$(x < 0$ or $x > 1)$ and $(x < -2$ or $x > 1)$ $\quad$ Hence $x < -2$ or $x > 1$	M1 A1 A1	7
4.	(a) $\Sigma\,(2r^2 - 6r) = \frac{1}{3}n(n+1)(2n+1) - 3n(n+1)$	M1 A1	
	$= \frac{1}{3}n(n+1)(2n+1-9) = \frac{2}{3}n(n+1)(n-4)$	M1 A1 A1	
	(b) $S_{30} - S_9 = 16\,120 - 300 = 15\,820$	M1 A1 A1	8
5.	Integrating factor $= e^{\int -1/x\,dx} = e^{-\ln x} = 1/x$ $\quad$ $\frac{1}{x}\frac{dy}{dx} - \frac{1}{x^2}y = \frac{1}{x}\ln 2x$	M1 A1 A1	
	$\frac{d}{dx}\left(\frac{y}{x}\right) = \frac{1}{x}\ln 2x$ $\quad$ $\frac{y}{x} = \int \frac{1}{x}\ln 2x\,dx = \frac{1}{2}(\ln 2x)^2 + c$	A1 M1 A1	
	$y = \frac{1}{2}x(\ln 2x)^2 + cx$ $\quad$ $y(1/2) = 1 : c = 2$ $\quad$ $y = \frac{1}{2}x(\ln 2x)^2 + 2x$	M1 A1 A1	9
6.	(a) $r^4 = 2a^2(r\cos\theta)(r\sin\theta)$ $\quad$ $r^2 = a^2\sin 2\theta$	M1 A1 A1	
	(b) Area $= \frac{a^2}{2}\int_0^{\pi/2}\sin 2\theta\,d\theta = -\frac{a^2}{4}[\cos 2\theta]_0^{\pi/2} = \frac{a^2}{2}$	M1 A1 M1 A1 A1	
	(c) $a = a\sec(\alpha - \pi/4)$ $\quad$ $\sec(\alpha - \pi/4) = 1$ $\quad$ $\alpha = \pi/4$	M1 A1 A1	11
7.	(a) $\lvert z\rvert = \lvert a + bi\rvert / \lvert 3 + 4i\rvert = [\sqrt{(a^2 + b^2)}]/5$	M1 A1 A1	
	(b) $z = [(3a + 4b) + i(3b - 4a)]/25$ $\quad$ If $\arg z = 3\pi/4$,	M1 A1	
	$\mathrm{Re}(z) = -\mathrm{Im}(z)$ $\quad$ $3a + 4b = 4a - 3b$ $\quad$ $a = 7b$	M1 A1 A1	
	(c) $a^2 + b^2 = 50$, so $50b^2 = 50$ $\quad$ $\mathrm{Re}(z) < 0$, so $b = -1$, $a = -7$	M1 A1 A1 A1	
	(d) Points shown at $-1 + i$, $-1 - i$	B1 B1	14
8.	(a) If $y = ue^x$, $\quad$ $\frac{dy}{dx} = e^x\left(u + \frac{du}{dx}\right)$ $\quad$ $\frac{d^2y}{dx^2} = e^x\left(u + 2\frac{du}{dx} + \frac{d^2u}{dx^2}\right)$	M1 A1 M1 A1	
	Eqn. is $(1+2x)\left(u + 2\frac{du}{dx} + \frac{d^2u}{dx^2}\right) + (3-4x)\left(u + \frac{du}{dx}\right) + 2(x-2)u = 0$	B1	
	Simplifying gives $(1+2x)\frac{d^2u}{dx^2} + 5\frac{du}{dx} = 0$	M1 A1	
	(b) $(1+2x)\frac{dv}{dx} + 5v = 0$ $\quad$ $\frac{1}{5v}dv = -\frac{1}{2x+1}dx$	M1 A1	
	$\frac{1}{5}\ln v = -\frac{1}{2}\ln(2x+1) + c$ $\quad$ $v = A(2x+1)^{-5/2}$	M1 A1 A1	
	(c) $u = p(2x+1)^{-3/2} + q$ $\quad$ $y = e^x(p(2x+1)^{-3/2} + q)$	M1 A1 A1	15