1. Find the quadratic equation, with real integer coefficients, which has $(5 + i)$ as one of its roots. **(4 marks)**

2. Find the set of values of x for which

$$|2x+3| > |3x-2|.$$ **(5 marks)**

3. (a) Show that there is a root of the equation $7 \tan x = 5x$ lying between 4 and 4·5. **(2 marks)**

   (b) Apply the Newton-Raphson method once to $f(x) = 7 \tan x - 5x$, with $x_1 = 4{\cdot}3$, to find an estimate of this root, correct to 1 decimal place. **(4 marks)**

4. (a) Find an expression in terms of n for $\sum_{r=1}^{n} (r + 2)^2$, giving your answer in factorised form. **(5 marks)**

   (b) Hence or otherwise evaluate $\sum_{r=1}^{17} (r^2 + 4r)$. **(3 marks)**

5. A quantity y is varying with time t, in such a way that the difference between y and its rate of change with respect to t is directly proportional to t.
   Given that, when $t = 1$, $y = 1$ and the rate of change of y with respect to t is 2,

   (a) formulate a differential equation relating y and t. **(3 marks)**

   (b) Solve this equation to find y as a function of t. **(8 marks)**

6. Given that $w = -6 + 8i$ and that $wz = 7 + 24i$,

   (a) find the real and imaginary parts of the complex number z. **(4 marks)**

   (b) Find the modulus and argument of z, giving the argument in radians to 1 decimal place. **(4 marks)**

   (c) If $kw^* + z^*$ is real, find the value of the real constant k and state the value of $kw^* + z^*$. **(4 marks)**

7. (a) Let $x = e^t$ and let y be a function of x. Given that $e^t \frac{dy}{dt} = \frac{dy}{dx}$,

show that $e^{2t} \frac{d^2y}{dx^2} = \frac{d^2y}{dt^2} - \frac{dy}{dt}$. **(5 marks)**

(b) Show that the substitution $x = e^t$ transforms the differential equation

$$2x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$$

into the differential equation

$$2\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + y = 0.$$

and hence find the general solution of the first equation. **(8 marks)**

8. The curve C has polar equation $r = a(3 + \cos\theta)$, where $a > 0$ and $0 \leq \theta < 2\pi$.

(a) Sketch the curve C. **(3 marks)**

(b) Calculate, in terms of a and π, the area enclosed by C. **(7 marks)**

The line $r = 3a \sec\theta$ cuts C at the point with polar coordinates (r_0, θ_0).

(c) Show that $0{\cdot}5 < \theta_0 < 1$. **(2 marks)**

(d) Use linear interpolation twice on the interval $(0{\cdot}5, 1)$ to obtain an estimate of θ_0 to two decimal places. **(4 marks)**