PURE MATHS 4 (A) TEST PAPER 10 : ANSWERS AND MARK SCHEME

1. $(x-5-\mathrm{i})(x-5+\mathrm{i})=0$ $\quad x^2-10x+26=0$ M1 A1 M1 A1 4

2. $(2x+3)^2>(3x-2)^2$ $\quad 5x^2-24x-5<0$ M1 A1
   $(5x+1)(x-5)<0$ $\quad -1/5<x<5$ M1 A1 A1 5

3. (a) Let $f(x)=7\tan x-5x$ $\quad f(4)=-11{\cdot}9,\ f(4{\cdot}5)=9{\cdot}96$ M1 A1
   (b) $f'(x)=7\sec^2 x-5$ $\quad 4{\cdot}3-f(4{\cdot}3)/f'(4{\cdot}3)=4{\cdot}4$ (1 d.p.) B2 M1 A1 6

4. (a) $\Sigma(r^2+4r+4)=\frac{1}{6}n(n+1)(2n+1)+2n(n+1)+4n$ M1 A1
   $=\frac{1}{6}n(2n^2+3n+1+12n+12+24)=\frac{1}{6}n(2n^2+15n+37)$ A1 M1 A1
   (b) $S_{17}-4(17)=2465-68=2397$ M1 A1 A1 8

5. (a) $\frac{dy}{dt}-y=kt$ $\quad t=1, y=1, y'=2: k=1$ $\quad \frac{dy}{dt}-y=t$ B1 M1 A1
   (b) I.F. $=e^{\int -1\,dt}=e^{-t}$ $\quad e^{-t}\frac{dy}{dt}-e^{-t}y=te^{-t}$ $\quad \frac{d}{dt}(e^{-t}y)=te^{-t}$ B1 M1 A1
   $e^{-t}y=\int te^{-t}\,dt=-te^{-t}-e^{-t}+c$ $\quad y=ce^{t}-t-1$ M1 A1 A1
   $y(1)=1:\ ce=3$ $\quad c=3e^{-1}$ $\quad y=3e^{t-1}-t-1$ M1 A1 11

6. (a) $z=\frac{7+24\mathrm{i}}{-6+8\mathrm{i}}=\frac{(7+24\mathrm{i})(-6-8\mathrm{i})}{100}=\frac{150-200\mathrm{i}}{100}=\frac{3}{2}-2\mathrm{i}$ M1 A1 A1 A1
   (b) $|z|=\sqrt{(9/4+4)}=5/2$ M1 A1
   arg (z) = arctan $(-4/3)$; in 3rd quadrant, so $-0{\cdot}93$ M1 A1
   (c) $k(-6-8\mathrm{i})+(1{\cdot}5-2\mathrm{i})$ real, so $-8k-2=0$ $\quad k=1/4$ $\quad$ Value $=0$ M1 M1 A1 A1 12

7. (a) $\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx}=\frac{d}{dt}\left(e^{-t}\frac{dy}{dt}\right)\frac{dt}{dx}$ M1 A1 A1
   $=e^{-t}\left(e^{-t}\frac{d^2y}{dt^2}-e^{-t}\frac{dy}{dt}\right)=e^{-2t}\left(\frac{d^2y}{dt^2}-\frac{dy}{dt}\right)$; hence result M1 A1
   (b) Eqn. is $2e^{2t}\frac{d^2y}{dx^2}-e^{t}\frac{dy}{dx}-y=0$ $\quad 2\left(\frac{d^2y}{dt^2}-\frac{dy}{dt}\right)-\frac{dy}{dt}-y=0$ M1 A1 A1
   (c) $2u^2-3u+1=0: u=1,\ 1/2$ $\quad y=ae^{t}+be^{t/2}=ax+b\sqrt{x}$ M1 A1 M1 A1 A1 13

8. (a) Curve sketched through $(4, 0)$, $(\pm 3, \pi/2)$, $(2, \pi)$ B3
   (b) Area $=\frac{a^2}{2}\int_0^{2\pi}(\cos^2\theta+6\cos\theta+9)\,d\theta$ M1 A1
   $=\frac{a^2}{4}\int_0^{2\pi}(\cos 2\theta+12\cos\theta+19)\,d\theta$ M1 A1 A1
   $=\frac{a^2}{8}[\sin 2\theta+24\sin\theta+38\theta]_0^{2\pi}=\frac{19\pi a^2}{2}$ M1 A1
   (c) Let $f(\theta)=a(3+\cos\theta-3\sec\theta)$ $\quad f(0{\cdot}5)=0{\cdot}46a,\ f(1)=-2{\cdot}01a$ M1 A1
   (d) $0{\cdot}5+0{\cdot}46a/2{\cdot}47a\times 0{\cdot}5=0{\cdot}593$ $\quad f(0{\cdot}593)=0{\cdot}212a$ M1 A1
   $0{\cdot}593+(0{\cdot}212a/2{\cdot}223a)\times(1-0{\cdot}593)=0{\cdot}63$ (to 2 d.p.) M1 A1 16