1. (a) Given that $a > 0$, sketch the curve with polar equation

$$r = a \sin 3\theta, \ 0 \leq \theta \leq \pi.$$ **(4 marks)**

(b) State the maximum distance of a point on this curve from the pole. **(1 mark)**

2. Find the complete solution set of the inequality

$$|x^2 + 2| < |x - 4|.$$ **(6 marks)**

3. Given that $f(x) = 2e^x - x^2 - 3$,

(a) show that there is a root of the equation $f(x) = 0$ between 0 and 1. **(2 marks)**

(b) Taking 0·5 as a first approximation, use the Newton-Raphson process once to obtain a second approximation to this root, correct to 3 significant figures. **(4 marks)**

4. Given that $w = 1 + i$ and $z = 1 - i\sqrt{3}$,

(a) find arg w and arg z. **(3 marks)**

(b) Express wz in the form $r(\cos\theta + i \sin\theta)$, where $r > 0$ and $-\pi \leq \theta < \pi$, giving the value of r in surd form and θ in terms of π. **(4 marks)**

(c) Hence or otherwise state the modulus and argument of $\frac{1}{wz}$. **(2 marks)**

5. (a) Show that

$$\sum_{r=1}^{n} (3r - 1)(3r + 2) = n(3n^2 + 6n + 1).$$ **(5 marks)**

(b) Using the result in (a), or otherwise, evaluate

$$\sum_{r=10}^{50} (9r^2 + 3r).$$ **(4 marks)**

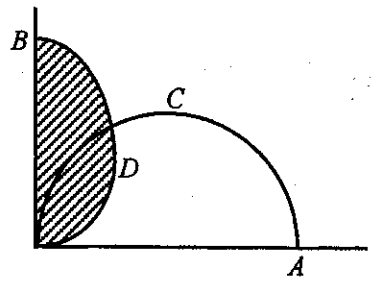
6. (a) Solve the differential equation

$$2x\frac{dy}{dx} - y = x,$$

given that $x \neq 0$ and that $y = -2$ when $x = 1$. **(8 marks)**

(b) Hence find the value of y when $x = 4$. **(2 marks)**

7. The diagram shows the curves with polar equations $r = 2\theta$ and $r = 5\cos\theta$, for $0 \leq \theta \leq \frac{\pi}{2}$.



(a) Calculate the polar coordinates of

(i) the points A and B, **(3 marks)**

(ii) the point C, where the tangent to $r = 5\cos\theta$ is parallel to the initial line. **(2 marks)**

(b) Show that at the point D, where the tangent to $r = 2\theta$ is perpendicular to the initial line,

$$\tan\theta = \frac{1}{\theta}.$$ **(4 marks)**

(c) Find, in terms of π, the area of the shaded region. **(5 marks)**

8. (a) Find the general solution of the differential equation

$$2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 2y = 25\cos x.$$ **(8 marks)**

(b) Find the particular solution of the above equation for which $y = 1$ and $\frac{dy}{dx} = -1$ when $x = 0$. **(5 marks)**

(c) Hence find, to 2 significant figures, the value of y when $x = \pi$. **(3 marks)**

