6672/01

Edexcel GCE

Pure Mathematics

Unit P2

Advanced Subsidiary / Advanced

Time: 1 hour 30 minutes

Materials required for the examination   Items included with these question papers

Answer Book (AB04)   Nil
Graph Paper (GP02)
Mathematical Formulae

Candidates may use any calculator EXCEPT those with a facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as Texas TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

Full marks may be obtained for answers to ALL questions.

In the boxes on the Answer Book provided, write the name of the Examining Body (Edexcel), your Centre Number, Candidate Number, the Unit Title (Pure Mathematics P2), the Paper Reference(6672), your surname, other names and signature.

Information for Candidates

A booklet 'Mathematical Formulae including Statistical Formulae and Tables' is provided.

This paper has 8 questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly numbered.

You must show sufficient working to make your methods clear to the Examiner. Answers without working will gain no credit

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1. An engineer estimated the area of the vertical cross-section of water flowing under a bridge. For her model she measured the depth of water at 4 m intervals from one end of the bridge to the other end. Her results are given in the table.

Distance from one end (m)	0	4	8	12	16	20	24
Depth (m)	1.2	2.3	3.8	4.9	3.2	1.9	0.6

She used the trapezium rule to estimate the area of the cross-section. Calculate the estimate she obtained. **(5 marks)**

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2. Express

$$\frac{5(x-3)(x+1)}{(x-12)(x+3)} - \frac{3(x+1)}{x-12}$$

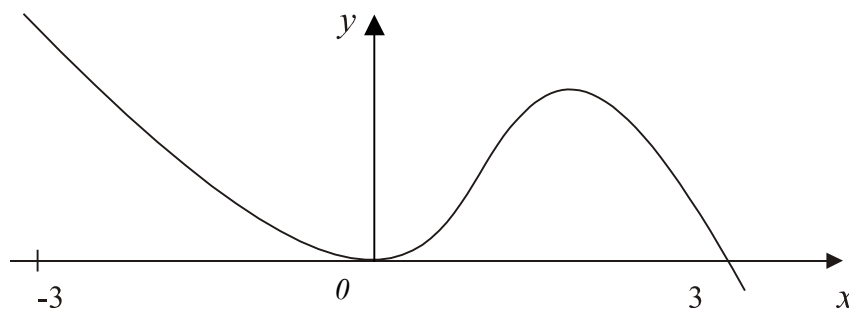as a single fraction in its simplest form. **(6 marks)**

3.

Fig. 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$.

In *separate* diagrams show, for $-3 \le x \le 3$, a sketch of the curve with equation

(a) $y = f(-x)$, **(2 marks)**

(b) $y = -f(x)$, **(2 marks)**

(c) $y = f(|x|)$, **(4 marks)**

marking on each sketch the *x*-coordinates of any point, or points, where a curve touches or crosses the *x*-axis.

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4. The finite region bounded by the curve with the equation $y = x - x^2$ and the x-axis is rotated through 360° about the x-axis. Using integration, find, in terms of π, the volume of its solid form.

(8 marks)

5. (a) By sketching the curves with equations $y = 4 - x^2$ and $y = e^x$, show that the equation $x^2 + e^x - 4 = 0$ has one negative root and one positive root. **(3 marks)**

(b) Use the iteration formula $x_{n+1} = -\sqrt{\left(4 - e^{x_n}\right)}$ with $x_0 = -2$ to find in turn x_1, x_2 and x_3 and hence write down an approximation to the negative root of the equation, giving your answer to 3 decimal places. **(4 marks)**

An attempt to evaluate the positive root of the equation is made using the iteration formula $x_{n+1} = \sqrt{\left(4 - e^{x_n}\right)}$ with $x_0 = 1.3$.

(c) Describe the result of such an attempt. **(2 marks)**
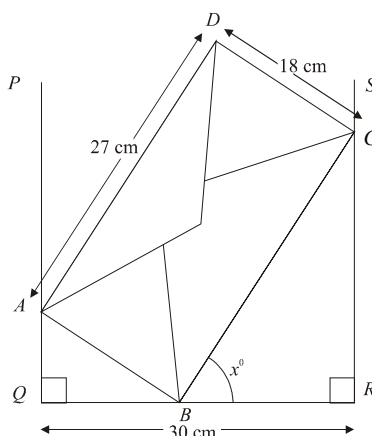
6.

Fig. 2

Figure 2 shows the rectangular cross-section $PQRS$ of a letter rack. A rectangular envelope $ABCD$ rests in the vertical plane $PQRS$ inside the letter rack. QR is horizontal. $QR = 30$ cm, $AD = 27$ cm and $CD = 18$ cm. The bottom edge, BC, of the envelope, makes an angle $x°$ with the base QR of the rack.

(a) Prove that $9\cos x° + 6\sin x° = 10$. **(4 marks)**

(b) Express $9\cos x° + 6\sin x°$ in the form $R\cos\left(x° - \alpha°\right)$, where $R > 0$ and $0 < \alpha < 90$, giving the values of R and α to 2 decimal places. **(4 marks)**

(c) Hence, or otherwise, find x, giving your answer to the nearest tenth of a degree. **(2 marks)**

7. The function f is given by

$$f : x \mapsto \ln(4 - 2x), \quad x \in \mathbb{R}, \quad x < 2.$$

(a) Find an expression for $f^{-1}(x)$. **(4 marks)**

(b) Sketch the curve with equation $y = f^{-1}(x)$, showing the coordinates of the points where the curve meets the axes. **(3 marks)**

The function g is given by

$$g : x \mapsto 3^x, \quad x \in \mathbb{R}.$$

(c) Find the value of x for which g(x) = 1.5, giving your answer to 3 decimal places.

(3 marks)

(d) Evaluate gf(1) to 3 decimal places. **(3 marks)**

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8. The curve with equation $y = \frac{1}{2}e^x$ meets the y-axis at the point A.

(a) Prove that the tangent at A to the curve has equation $2y = x + 1$. **(4 marks)**

The point B has x-coordinate ln 4 and lies on the curve. The normal at B to the curve meets the tangent at A to the curve at the point C.

(b) Prove that the x-coordinate of C is $\frac{3}{2} + \ln 2$ and find the y-coordinate of C. **(8 marks)**

(c) Find, in terms of e, the area of the finite region bounded by the curve with equation $y = \frac{1}{2}e^x$, the coordinate axes and the line with equation $x + 2 = 0$. **(4 marks)**

END