Paper Reference(s)

6681

Edexcel GCE

Mechanics M5

Advanced Level

Specimen Paper

Time: 1 hour 30 minutes

Materials required for examination
Answer Book (AB16)
Mathematical Formulae (Lilac)
Graph Paper (ASG2)

Items included with question papers
Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M5), the paper reference (6681), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has eight questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. A bead of mass 0.125 kg is threaded on a smooth straight horizontal wire. The bead moves from rest at the point A with position vector $(2\mathbf{i} + \mathbf{j} - \mathbf{k})$ m relative to a fixed origin O to a point B with position vector $(3\mathbf{i} - 4\mathbf{j} - \mathbf{k})$ m relative to O under the action of a force $\mathbf{F} = (14\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$ N. Find

(a) the work done by $\mathbf{F}$ as the bead moves from A to B,

(3)

(b) the speed of the bead at B.

(2)

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2. (a) Prove, using integration, that the moment of inertia of a uniform rod, of mass m and length $2a$, about an axis perpendicular to the rod through its centre is $\frac{1}{3}ma^2$.

(3)

A uniform wire of mass $4m$ and length $8a$ is bent into the shape of a square.

(b) Find the moment of inertia of the square about the axis through the centre of the square perpendicular to its plane.

(4)

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3. Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ and a couple $\mathbf{G}$ act on a rigid body. The force $\mathbf{F}_1 = (3\mathbf{i} + 4\mathbf{j})$ N acts through the point with position vector $2\mathbf{i}$ m relative to a fixed origin O. The force $\mathbf{F}_2 = (2\mathbf{i} - \mathbf{j} + \mathbf{k})$ N acts through the point with position vector $(\mathbf{i} + \mathbf{j})$ m relative to O. The forces and couple are equivalent to a single force $\mathbf{F}$ acting through O.

(a) Find $\mathbf{F}$.

(2)

(b) Find $\mathbf{G}$.

(5)

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4. A uniform circular disc, of mass $2m$ and radius a, is free to rotate in a vertical plane about a fixed, smooth horizontal axis through a point of its circumference. The axis is perpendicular to the plane of the disc. The disc hangs in equilibrium. A particle P of mass m is moving horizontally in the same plane as the disc with speed $\sqrt{(20ag)}$. The particle strikes, and adheres to, the disc at one end of its horizontal diameter.

(a) Find the angular speed of the disc immediately after P strikes it.

(7)

(b) Verify that the disc will turn through an angle of 90° before first coming to instantaneous rest.

(3)

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5. A uniform square lamina $ABCD$ of side a and mass m is free to rotate in vertical plane about a horizontal axis through A. The axis is perpendicular to the plane of the lamina. The lamina is released from rest when $t = 0$ and AC makes a small angle with the downward vertical through A.

(a) Show that the moment of inertia of the lamina about the axis is $\frac{2}{3}ma^2$.

(3)

(b) Show that the motion of the lamina is approximately simple harmonic.

(5)

(c) Find the time t when AC is first vertical.

(2)

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6. A uniform rod AB of mass m and length $4a$ is free to rotate in a vertical plane about a horizontal axis through the point O of the rod, where $OA = a$. The rod is slightly disturbed from rest when B is vertically above A.

(a) Find the magnitude of the angular acceleration of the rod when it is horizontal.

(4)

(b) Find the angular speed of the rod when it is horizontal.

(2)

(c) Calculate the magnitude of the force acting on the rod at O when the rod is horizontal.

(5)

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7. As a hailstone falls under gravity in still air, its mass increases. At time t the mass of the hailstone is m. The hailstone is modelled as a uniform sphere of radius r such that

$$\frac{\mathrm{d}r}{\mathrm{d}t} = kr \ ,$$

where k is a positive constant.

(*a*) Show that $\frac{\mathrm{d}m}{\mathrm{d}t} = 3km$.

(2)

Assuming that there is no air resistance,

(*b*) show that the speed v of the hailstone at time t satisfies

$$\frac{\mathrm{d}v}{\mathrm{d}t} = g - 3kv \ .$$

(4)

Given that the speed of the hailstone at time $t = 0$ is u,

(c) find an expression for v in terms of t.

(5)

(*d*) Hence show that the speed of the hailstone approaches the limiting value $\frac{g}{3k}$.

(1)

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8. A particle P moves in the x-y plane and has position vector $\mathbf{r}$ metres relative to a fixed origin O at time t s. Given that $\mathbf{r}$ satisfies the vector differential equation

$$\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} + 9\mathbf{r} = 8\sin t\,\mathbf{i}$$

and that when $t = 0$ s, P is at O and moving with velocity $(\mathbf{i} + 3\mathbf{j})$ m s^{-1},

(a) find $\mathbf{r}$ at time t.

(11)

(b) Hence find when P next returns to O.

(2)

END