Paper Reference(s)

6677

Edexcel GCE

Mechanics M1

Advanced Subsidiary

Specimen Paper

Time: 1 hour 30 minutes

Materials required for examination
Answer Book (AB16)
Mathematical Formulae (Lilac)
Graph Paper (ASG2)

Items included with question papers
Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M1), the paper reference (6677), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

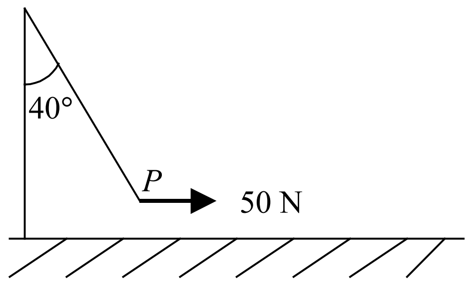
Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. **Figure 1**



A tennis ball P is attached to one end of a light inextensible string, the other end of the string being attached to a the top of a fixed vertical pole. A girl applies a horizontal force of magnitude 50 N to P, and P is in equilibrium under gravity with the string making an angle of 40° with the pole, as shown in Fig. 1.

By modelling the ball as a particle find, to 3 significant figures,

(*a*) the tension in the string,

(3)

(*b*) the weight of P.

(4)

---

2. A car starts from rest at a point O and moves in a straight line. The car moves with constant acceleration 4 m s^{-2} until it passes the point A when it is moving with speed 10 m s^{-1}. It then moves with constant acceleration 3 m s^{-2} for 6 s until it reaches the point B. Find

(*a*) the speed of the car at B,

(2)

(*b*) the distance OB.

(5)

---

3. **Figure 2**

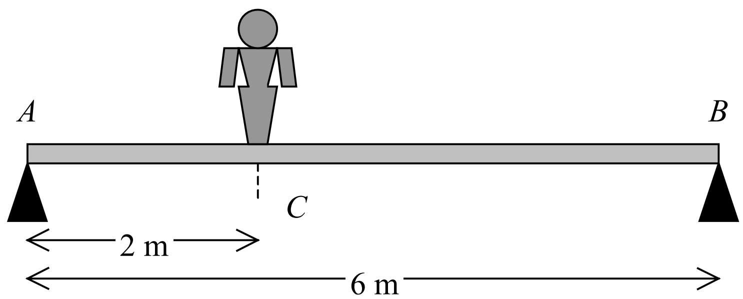


Fig. 2

A non-uniform plank of wood AB has length 6 m and mass 90 kg. The plank is smoothly supported at its two ends A and B, with A and B at the same horizontal level. A woman of mass 60 kg stands on the plank at the point C, where $AC = 2$ m, as shown in Fig. 2. The plank is in equilibrium and the magnitudes of the reactions on the plank at A and B are equal. The plank is modelled as a non-uniform rod and the woman as a particle. Find

(*a*) the magnitude of the reaction on the plank at B,

(2)

(*b*) the distance of the centre of mass of the plank from A.

(5)

(*c*) State briefly how you have used the modelling assumption that

(i) the plank is a rod,

(ii) the woman is a particle.

(2)

4. A train T, moves from rest at Station A with constant acceleration 2 m s^{-2} until it reaches a speed of 36 m s^{-1}. In maintains this constant speed for 90 s before the brakes are applied, which produce constant retardation 3 ms^{-2}. The train T_1 comes to rest at station B.

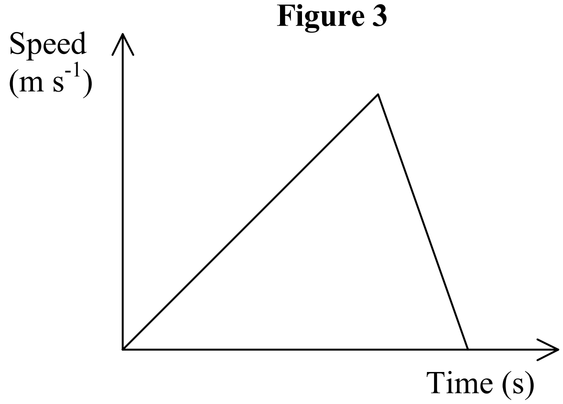
(*a*) Sketch a speed-time graph to illustrate the journey of T_1 from A to B.

(3)

(*b*) Show that the distance between A and B is 3780 m.

(5)

Figure 3



A second train T_2 takes 150 s to move form rest at A to rest at B. Figure 3 shows the speed-time graph illustrating this journey.

(*c*) Explain briefly one way in which T_1's journey differs from T_2's journey.

(1)

(*d*) Find the greatest speed, in m s^{-1}, attained by T_2 during its journey.

(3)

5. A truck of mass 3 tonnes moves on straight horizontal rails. It collides with truck B of mass 1 tonne, which is moving on the same rails. Immediately before the collision, the speed of A is 3 m s^{-1}, the speed of B is 4 m s^{-1}, and the trucks are moving towards each other. In the collision, the trucks couple to form a single body C, which continues to move on the rails.

(*a*) Find the speed and direction of C after the collision.

(4)

(*b*) Find, in Ns, the magnitude of the impulse exerted by B on A in the collision.

(3)

(*c*) State a modelling assumption which you have made about the trucks in your solution

(1)

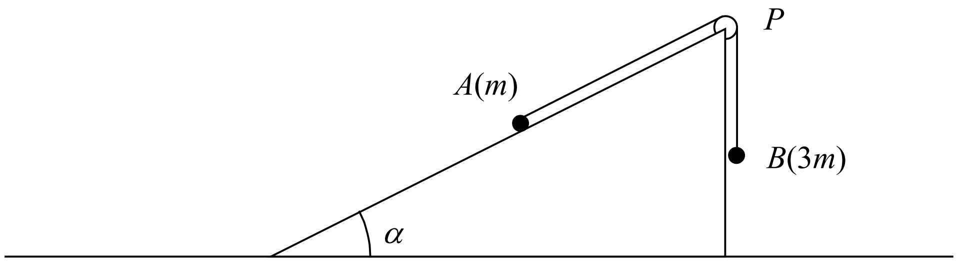
Immediately after the collision, a constant braking force of magnitude 250 N is applied to C. It comes to rest in a distance d metres.

(*d*) Find the value of d.

(4)

6.

Figure 4



A particle of mass m rests on a rough plane inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$. The particle is attached to one end of a light inextensible string which lies in a line of greatest slope of the plane and passes over a small light smooth pulley P fixed at the top of the plane. The other end of the string is attached to a particle B of mass $3m$, and B hangs freely below P, as shown in Fig. 4. The particles are released from rest with the string taut. The particle B moves down with acceleration of magnitude $\frac{1}{2}g$. Find

(*a*) the tension in the string,

(4)

(*b*) the coefficient of friction between A and the plane.

(9)

---

7. Two cars A and B are moving on straight horizontal roads with constant velocities. The velocity of A is 20 m s^{-1} due east, and the velocity of B is $(10\mathbf{i} + 10\mathbf{j})$ m s^{-1}, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors directed due east and due north respectively. Initially A is at the fixed origin O, and the position vector of B is $300\mathbf{i}$ m relative to O. At time t seconds, the position vectors of A and B are $\mathbf{r}$ metres and $\mathbf{s}$ metres respectively.

(*a*) Find expressions for $\mathbf{r}$ and $\mathbf{s}$ in terms of t. **(3)**

(*b*) Hence write down an expression for $\overrightarrow{AB}$ in terms of t. **(1)**

(*c*) Find the time when the bearing of B from A is 045°. **(5)**

(*d*) Find the time when the cars are again 300 m apart. **(6)**

END

Paper Reference(s)

6678

Edexcel GCE

Mechanics M2

Advanced Level

Specimen Paper

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Answer Book (AB16)	Nil
Mathematical Formulae (Lilac)	
Graph Paper (ASG2)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has eight questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. The vectors **i** and **j** are perpendicular unit vectors in a horizontal plane. A ball of mass 0.5 kg is moving with velocity $-20\mathbf{i}$ m s^{-1} when it is struck by a bat. The bat gives the ball an impulse of $(15\mathbf{i} + 10\mathbf{j})$ Ns.

Find, to 3 significant figures, the speed of the ball immediately after it has been struck.

(5)

---

2. A bullet of mass 6 grams passes horizontally through a fixed, vertical board. After the bullet has travelled 2 cm through the board its speed is reduced from 400 m s^{-1} to 250 m s^{-1}. The board exerts a constant resistive force on the bullet.

Find, to 3 significant figures, the magnitude of this resistive force.

(5)

---

3. At time t seconds, a particle P has position vector $\mathbf{r}$ metres relative to a fixed origin O, where

$$\mathbf{r} = (t^3 - 3t)\mathbf{i} + 4t^2\mathbf{j}, \quad t \geq 0.$$

Find

(*a*) the velocity of P at time t seconds,

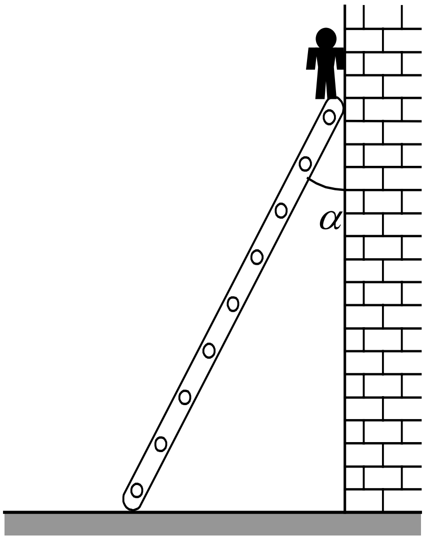
(2)

(*b*) the time when P is moving parallel to the vector $\mathbf{i} + \mathbf{j}$.

(5)

---

4. **Figure 1**



A uniform ladder, of mass m and length $2a$, has one end on rough horizontal ground. The other end rests against a smooth vertical wall. A man of mass $3m$ stands at the top of the ladder and the ladder is in equilibrium. The coefficient of friction between the ladder and the ground is $\frac{1}{4}$, and the ladder makes an angle α with the vertical, as shown in Fig. 1. The ladder is in a vertical plane perpendicular to the wall.

Show that $\tan\alpha \leq \frac{2}{7}$.

(9)

---

5. A straight road is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{20}$. A lorry of mass 4800 kg moves up the road at a constant speed of 12 m s^{-1}. The non-gravitational resistance to the motion of the lorry is constant and has magnitude 2000 N.

(*a*) Find, in kW to 3 significant figures, the rate of working of the lorry's engine.

(5)

The road becomes horizontal. The lorry's engine continues to work at the same rate and the resistance to motion remains the same.

Find

(*b*) the acceleration of the lorry immediately after the road becomes horizontal,

(3)

(*c*) the maximum speed, in m s^{-1} to 3 significant figures, at which the lorry will go along the horizontal road.

(3)

---

6. A cricket ball is hit from a height of 0.8 m above horizontal ground with a speed of 26 m s^{-1} at an angle α above the horizontal, where $\tan \alpha = \frac{5}{12}$. The motion of the ball is modelled as that of a particle moving freely under gravity.

(*a*) Find, to 2 significant figures, the greatest height above the ground reached by the ball.

(4)

When the ball has travelled a horizontal distance of 36 m, it hits a window.

(*b*) Find, to 2 significant figures, the height above the ground at which the ball hits the window.

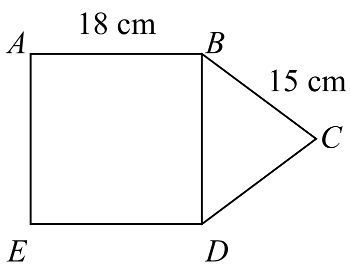
(7)

(*c*) State one physical factor which could be taken into account in any refinement of the model which would make it more realistic.

(1)

Figure 2

7.



A uniform plane lamina *ABCDE* is formed by joining a uniform square *ABDE* with a uniform triangular lamina *BCD*, of the same material, along the side *BD*, as shown in Fig. 2. The lengths *AB*, *BC* and *CD* are 18 cm, 15 cm and 15 cm respectively.

(a) Find the distance of the centre of mass of the lamina from *AE*.

(9)

The lamina is freely suspended from *B* and hangs in equilibrium.

(*b*) Find, in degrees to one decimal place, the angle which *BD* makes with the vertical.

(4)

---

8. A particle A of mass m is moving with speed $3u$ on a smooth horizontal table when it collides directly with a particle B of mass $2m$ which is moving in the opposite direction with speed u. The direction of motion of A is reversed by the collision. The coefficient of restitution between A and B is e.

(*a*) Show that the speed of B immediately after the collision is $\frac{1}{3}(1 + 4e)u$.

(6)

(*b*) Show that $e > \frac{1}{8}$.

(3)

Subsequently B hits a wall fixed at right angles to the line of motion of A and B. The coefficient of restitution between B and the wall is $\frac{1}{2}$. After B rebounds from the wall, there is a further collision between A and B.

(*c*) Show that $e < \frac{1}{4}$.

(4)

END

Paper Reference(s)

6679

Edexcel GCE

Mechanics M3

Advanced Level

Specimen Paper

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Answer Book (AB16)	Nil
Mathematical Formulae (Lilac)	
Graph Paper (ASG2)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679), your surname, other name and signature.
Whenever a numerical value of g is required, take g = 9.8 m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

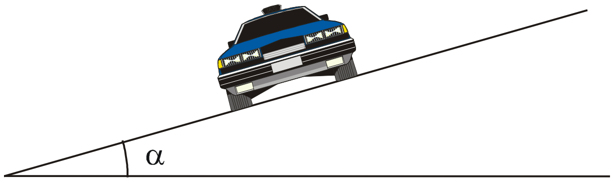
Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. **Figure 1**



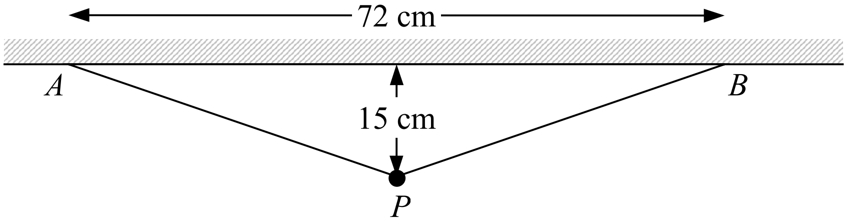
A car moves round a bend in a road which is banked at an angle α to the horizontal, as shown in Fig. 1. The car is modelled as a particle moving in a horizontal circle of radius 100 m. When the car moves at a constant speed of 14 m s^{-1}, there is no sideways frictional force on the car.

Find, in degrees to one decimal place, the value of α.

(7)

---

2. **Figure 2**



Two elastic ropes each have natural length 30 cm and modulus of elasticity λ N. One end of each rope is attached to a lead weight P of mass 2 kg and the other ends are attached to two points A and B on a horizontal ceiling, where AB = 72 cm. The weight hangs in equilibrium 15 cm below the ceiling, as shown in Fig. 2. By modelling P as a particle and the ropes as light elastic strings,

(*a*) find, to one decimal place, the value of λ .

(6)

(*b*) State how you have used the fact that P is modelled as a particle.

(1)

---

3. A particle P of mass 0.5 kg moves away from the origin O along the positive x-axis under the action of a force directed towards O of magnitude $\frac{2}{x^2}$ N, where $OP = x$ metres. When $x = 1$, the speed of P is 3 m s^{-1}. Find the distance of P from O when its speed has been reduced to 1.5 m s^{-1}.

(8)

---

4. A man of mass 75 kg is attached to one end of a light elastic rope of natural length 12 m. The other end of the rope is attached to a point on the edge of a horizontal ledge 19 m above the ground. The man steps off the ledge and falls vertically under gravity. The man is modelled as a particle falling from rest. He is brought to instantaneous rest by the rope when he is 1 m above the ground.
Find

(*a*) the modulus of elasticity of the rope,

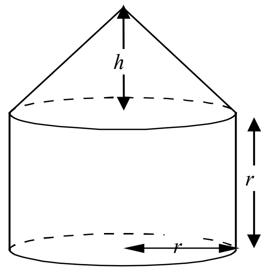
(5)

(*b*) the speed of the man when he is 2 m above the ground, giving your answer in m s^{-1} to 3 significant figures.

(5)

5.

Figure 3



A uniform solid, *S*, is placed with its plane face on horizontal ground. The solid consists of a right circular cylinder, of radius r and height r, joined to a right circular cone, of radius r and height h. The plane face of the cone coincides with one of the plane faces of the cylinder, as shown in Fig. 3.

(*a*) Show that the distance of the centre of mass of *S* from the ground is

$$\frac{6r^2 + 4rh + h^2}{4(3r + h)}.$$

(8)

The solid is now placed with its plane face on a rough plane which is inclined at an angle α to the horizontal. The plane is rough enough to prevent *S* from sliding. Given that $h = 2r$, and that *S* is on the point of toppling,

(*b*) find, to the nearest degree, the value of α.

(5)

---

6. A particle P is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is hanging in equilibrium below O when it receives a horizontal impulse giving it a speed u, where $u^2 = 3ga$. The string becomes slack when P is at the point B. The line OB makes an angle θ with the upward vertical.

(*a*) Show that $\cos\theta = \frac{1}{3}$.

(9)

(*b*) Show that the greatest height of P above B in the subsequent motion is $\dfrac{4a}{27}$.

(6)

---

7. A particle P of mass m is attached to one end of a light elastic string of natural length a and modulus of elasticity $6mg$. The other end of the string is attached to a fixed point O. When the particle hangs in equilibrium with the string vertical, the extension of the string is e.

(*a*) Find e.

(2)

The particle is now pulled down a vertical distance $\frac{1}{3}a$ below its equilibrium position and released from rest. At time t after being released, during the time when the string remains taut, the extension of the string is $e + x$. By forming a differential equation for the motion of P while the string remains taut,

(*b*) show that during this time P moves with simple harmonic motion of period $2\pi\sqrt{\dfrac{a}{6g}}$.

(6)

(*c*) Show that, while the string remains taut, the greatest speed of P is $\frac{1}{3}\sqrt{(6ga)}$.

(2)

(*d*) Find t when the string becomes slack for the first time.

(5)

END

Paper Reference(s)

6680

Edexcel GCE

Mechanics M4

Advanced Level

Specimen Paper

Time: 1 hour 30 minutes

Materials required for examination
Answer Book (AB16)
Mathematical Formulae (Lilac)
Graph Paper (ASG2)

Items included with question papers
Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M4), the paper reference (6680), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions.

Advice to Candidates

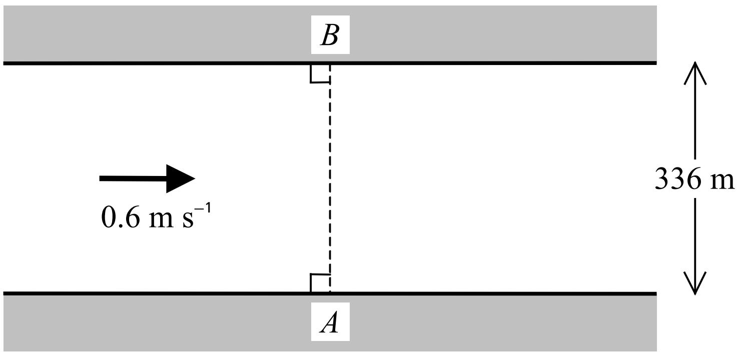
You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. A particle P of mass 2 kg moves in a straight line along a smooth horizontal plane. The only horizontal force acting on P is a resistance of magnitude $4v$ N, where v m s^{-1} is its speed. At time $t = 0$ s, P has a speed of 5 m s^{-1}. Find v in terms of t.

(6)

2.

Figure 1



A girl swims in still water at 1 m s^{-1}. She swims across a river which is 336 m wide and is flowing at 0.6 m s^{-1}. She sets off from a point A on one bank and lands at a point B, which is directly opposite A, on the other bank as shown in Fig. 1. Find

(*a*) the direction, relative to the earth, in which she swims,

(3)

(*b*) the time that she takes to cross the river.

(3)

---

3. A ball of mass m is thrown vertically upwards from the ground. When its speed is v the magnitude of the air resistance is modelled as being mkv^2, where k is a positive constant. The ball is projected with speed $\sqrt{\dfrac{g}{k}}$.

By modelling the ball as a particle,

(*a*) find the greatest height reached by the ball.

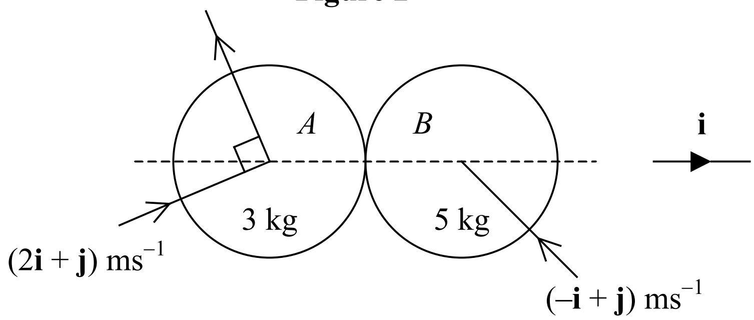
(9)

(*b*) State one physical factor which is ignored in this model.

(1)

---

4. **Figure 2**



Two smooth uniform spheres A and B, of equal radius, are moving on a smooth horizontal plane. Sphere A has mass 3 kg and velocity (2**i** + **j**) m s^{-1}, and sphere B has mass 5 kg and velocity (−**i** + **j**) m s^{-1}. When the spheres collide the line joining their centres is parallel to **i**, as shown in Fig. 2.

Given that the direction of A is deflected through a right angle by the collision, find

(a) the velocity of A after the collision,

(5)

(b) the coefficient of restitution between the spheres.

(6)

---

5. An elastic string spring of modulus $2mg$ and natural length l is fixed at one end. To the other end is attached a mass m which is allowed to hang in equilibrium. The mass is then pulled vertically downwards through a distance l and released from rest. The air resistance is modelled as having magnitude $2m\omega v$, where v is the speed of the particle and $\omega = \sqrt{\dfrac{g}{l}}$. The particle is at distance x from its equilibrium position at time t.

(a) Show that $\dfrac{\mathrm{d}^2 x}{\mathrm{d}t^2} + 2\omega\dfrac{\mathrm{d}x}{\mathrm{d}t} + 2\omega^2 x = 0$.

(7)

(b) Find the general solution of this differential equation.

(4)

(c) Hence find the period of the damped harmonic motion.

(1)

---

6. Two horizontal roads cross at right angles. One is directed from south to north, and the other from east to west. A tractor travels north on the first road at a constant speed of 6 m s^{-1} and at noon is 200 m south of the junction. A car heads west on the second road at a constant speed of 24 m s^{-1} and at noon is 960 m east of the junction.

(*a*) Find the magnitude and direction of the velocity of the car relative to the tractor.

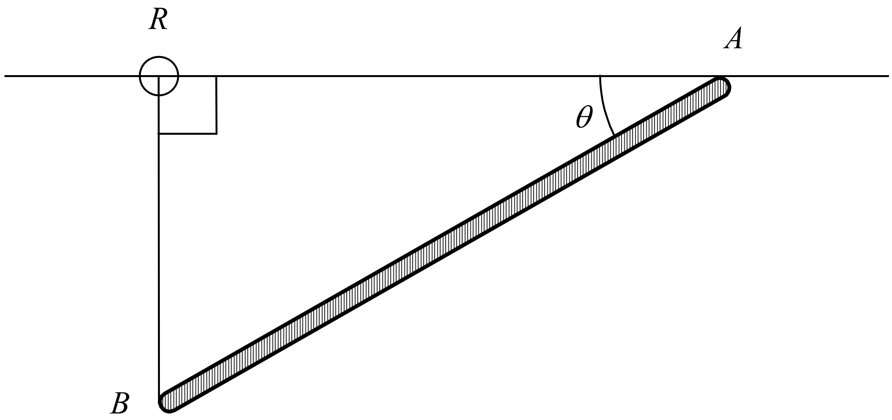
(6)

(*b*) Find the shortest distance between the car and the tractor.

(8)

7.

Figure 3



A uniform rod AB has mass m and length $2a$. The end A is smoothly hinged at a fixed point on a fixed straight horizontal wire. A smooth light ring R is threaded on the wire. The ring R is attached by a light elastic string, of natural length a and modulus of elasticity mg, to the end B of the rod. The end B is always vertically below R and angle $\angle RAB = \theta$, as shown in Fig. 3.

(*a*) Show that the potential energy of the system is

$$mga\left(2\sin^2\theta - 3\sin\theta\right) + \text{constant}.$$

(6)

(*b*) Hence determine the value of θ, $\theta < \frac{\pi}{2}$, for which the system is in equilibrium.

(5)

(c) Determine whether this position of equilibrium is stable or unstable.

(5)

END

Paper Reference(s)

6681

Edexcel GCE

Mechanics M5

Advanced Level

Specimen Paper

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Answer Book (AB16)	Nil
Mathematical Formulae (Lilac)	
Graph Paper (ASG2)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M5), the paper reference (6681), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has eight questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. A bead of mass 0.125 kg is threaded on a smooth straight horizontal wire. The bead moves from rest at the point A with position vector $(2\mathbf{i} + \mathbf{j} - \mathbf{k})$ m relative to a fixed origin O to a point B with position vector $(3\mathbf{i} - 4\mathbf{j} - \mathbf{k})$ m relative to O under the action of a force $\mathbf{F} = (14\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$ N. Find

(*a*) the work done by **F** as the bead moves from A to B,

(3)

(*b*) the speed of the bead at B.

(2)

---

2. (*a*) Prove, using integration, that the moment of inertia of a uniform rod, of mass m and length $2a$, about an axis perpendicular to the rod through its centre is $\frac{1}{3}ma^2$.

(3)

A uniform wire of mass $4m$ and length $8a$ is bent into the shape of a square.

(*b*) Find the moment of inertia of the square about the axis through the centre of the square perpendicular to its plane.

(4)

---

3. Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ and a couple **G** act on a rigid body. The force $\mathbf{F}_1 = (3\mathbf{i} + 4\mathbf{j})$ N acts through the point with position vector $2\mathbf{i}$ m relative to a fixed origin O. The force $\mathbf{F}_2 = (2\mathbf{i} - \mathbf{j} + \mathbf{k})$ N acts through the point with position vector $(\mathbf{i} + \mathbf{j})$ m relative to O. The forces and couple are equivalent to a single force **F** acting through O.

(*a*) Find **F**.

(2)

(*b*) Find **G**.

(5)

---

4. A uniform circular disc, of mass $2m$ and radius a, is free to rotate in a vertical plane about a fixed, smooth horizontal axis through a point of its circumference. The axis is perpendicular to the plane of the disc. The disc hangs in equilibrium. A particle P of mass m is moving horizontally in the same plane as the disc with speed $\sqrt{(20ag)}$. The particle strikes, and adheres to, the disc at one end of its horizontal diameter.

(*a*) Find the angular speed of the disc immediately after P strikes it.

(7)

(*b*) Verify that the disc will turn through an angle of 90° before first coming to instantaneous rest.

(3)

---

5. A uniform square lamina *ABCD* of side a and mass m is free to rotate in vertical plane about a horizontal axis through A. The axis is perpendicular to the plane of the lamina. The lamina is released from rest when $t = 0$ and AC makes a small angle with the downward vertical through A.

(*a*) Show that the moment of inertia of the lamina about the axis is $\frac{2}{3}ma^2$.

(3)

(*b*) Show that the motion of the lamina is approximately simple harmonic.

(5)

(*c*) Find the time t when AC is first vertical.

(2)

---

6. A uniform rod AB of mass m and length $4a$ is free to rotate in a vertical plane about a horizontal axis through the point O of the rod, where $OA = a$. The rod is slightly disturbed from rest when B is vertically above A.

(*a*) Find the magnitude of the angular acceleration of the rod when it is horizontal.

(4)

(*b*) Find the angular speed of the rod when it is horizontal.

(2)

(*c*) Calculate the magnitude of the force acting on the rod at O when the rod is horizontal.

(5)

---

7. As a hailstone falls under gravity in still air, its mass increases. At time t the mass of the hailstone is m. The hailstone is modelled as a uniform sphere of radius r such that

$$\frac{\mathrm{d}r}{\mathrm{d}t} = kr \ ,$$

where k is a positive constant.

(a) Show that $\dfrac{\mathrm{d}m}{\mathrm{d}t} = 3km$.

(2)

Assuming that there is no air resistance,

(b) show that the speed v of the hailstone at time t satisfies

$$\frac{\mathrm{d}v}{\mathrm{d}t} = g - 3kv \ .$$

(4)

Given that the speed of the hailstone at time $t = 0$ is u,

(c) find an expression for v in terms of t.

(5)

(d) Hence show that the speed of the hailstone approaches the limiting value $\dfrac{g}{3k}$.

(1)

---

8. A particle P moves in the x-y plane and has position vector $\mathbf{r}$ metres relative to a fixed origin O at time t s. Given that $\mathbf{r}$ satisfies the vector differential equation

$$\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} + 9\mathbf{r} = 8\sin t\ \mathbf{i}$$

and that when $t = 0$ s, P is at O and moving with velocity $(\mathbf{i} + 3\mathbf{j})$ m s^{-1},

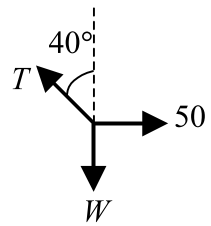
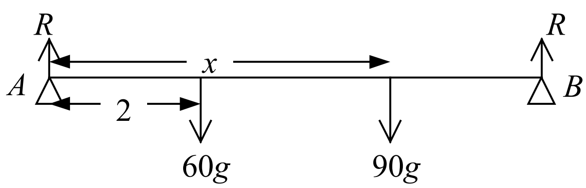
(a) find $\mathbf{r}$ at time t.

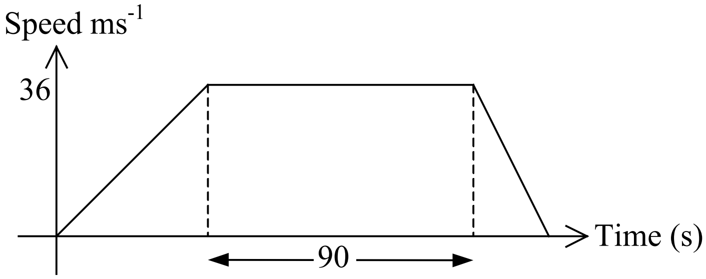
(11)

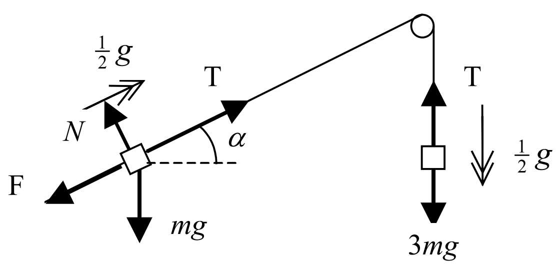
(b) Hence find when P next returns to O.

(2)

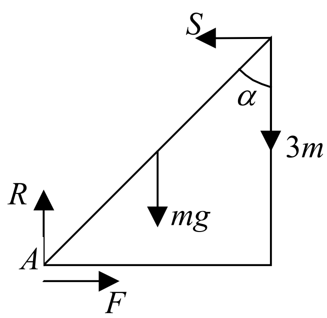
END

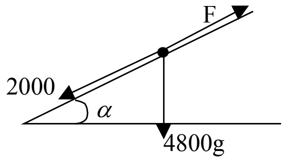
Question number	Scheme	Marks
1. (*a*)	 $T\sin 40° = 50 \Rightarrow, T \approx 77.8$ N	M1 A1, A1 (3)
(*b*)	$(\uparrow)$ $W = T\cos 40°$	M1 A1
	$W = 77.8\cos 40° \approx 59.6$ N	M1 A1 (4)
		(7 marks)
2. (*a*)	"$v = u + at$" $v_B = 10 + 3 \times 6 = 28$ m s^{-1}	M1A1 (2)
(*b*)	OA: "$v^2 = u^2 + 2as$" $10^2 = 0 + 2 \times 4 \times OA \Rightarrow OA = 12.5$ m	M1A1
	AB: "$s = ut + \frac{1}{2}at^2$" $\text{OB} = 10 \times 6 + \frac{1}{2} \times 3 \times 36 = 114$ m	M1A1
	$OB = 12.5 + 114 = 126.5$ m	A1 ft (5)
		(7 marks)
3.		
(*a*)	$R(\uparrow)$ $R + R = 60\text{g} + 90\text{g}$	
	$R = 75\text{g} = 735$ N	M1 A1 (2)
(*b*)	$\text{M}(A)$ $60g.2 + 90g.x = 75g.6$	M1 A1 A1 ft
	$90x = 450 - 120 = 330$	
	$x = \underline{3\tfrac{2}{3}\text{ m}}$ accept AWRT 3.67	M1 A1 (5)
(*c*)(i)	Plank remains a straight line/rigid.	B1
(ii)	Weight of woman acts at *C*.	B1 (2)
		(9 marks)

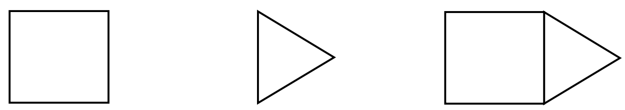
Question number	Scheme	Marks
4. (*a*)	 Shape	M1A1
	36,90	B1 (3)
(*b*)	Time to accelerate : time to decelerate	M1 A1
	18 s 12 s	
	Distance = area under graph	
	$= \frac{1}{2} \times 36 \times (90 + 120)$ m	M1 A1
	= 3780 m	A1 (5)
(*c*)	There is no period of constant maximum velocity	
	(OR "it speeds up and then immediately slows down again" OR "it attains a greater maximum speed")	B1 (1)
(*d*)	Let greatest speed be V m s^{-1} then	
	$\frac{1}{2} \times 150 \times V_{max} = 3780$	M1 A1
	$V = 50.4$	A1
		(9 marks)

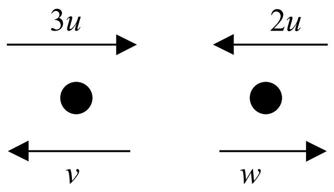
Question number	Scheme	Marks
5. (*a*)	Conservation of linear momentum applied	
	$3000 \times 3 - 4 \times 1000 = 4000 \times V$	M1 A1
	$V = 1.25$	A1
	Direction AB	A1 (4)
(*b*)	Impulse $= 3000\,[3 - 1.25]$ Ns	M1 A1 ft
	$= 5250$ Ns	A1 (3)
(*c*)	Trucks are assumed to be particles	B1 (1)
(*d*)	$F = ma \Rightarrow 250 = 4000\,a$	M1
	$a = \dfrac{1}{16}$	A1
	$v^2 = u^2 + 2as \Rightarrow 0 = (1.25)^2 - 2\left(\dfrac{1}{16}\right) d$	M1
	$d = 12.5$	A1 (4)
		(12 marks)
6.		
(*a*)	B: $3mg - T = 3m.\frac{1}{2}g$	M1 A1
	$T = \underline{\frac{3}{2}mg}$	M1 A1 (4)
(*b*)	A: $T - F - mg.\frac{3}{5} = m.\frac{1}{2}g$	M1 A1 A1
	$\Rightarrow F = \frac{2}{5}mg$	M1 A1 ft
	$N = mg.\frac{4}{5}$	M1 A1
	$\mu = \dfrac{F}{N} = \frac{1}{2}$	M1 A1 (9)
		(13 marks)

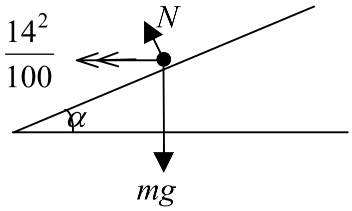
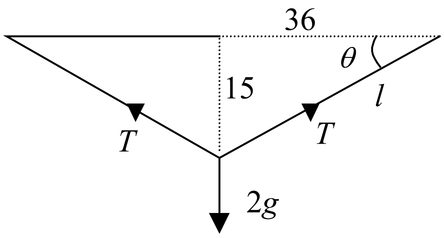
Question number	Scheme	Marks
7. (*a*)	$\mathbf{r} = 20\,t\mathbf{i}$	B1
	$\mathbf{s} = (300 + 10t)\mathbf{i} + (10t)\mathbf{j}$	M1 A1 (3)
(*b*)	$\overrightarrow{AB} = \mathbf{s} - \mathbf{r} = (300 - 10t)\mathbf{i} + (10t)\mathbf{j}$	B1 ft (1)
(*c*)	Bearing of B from A 045° $\Rightarrow \overrightarrow{AB} \,//^{e}\, \mathbf{i} + \mathbf{j}$	M1
	$\Rightarrow \dfrac{10t}{300 - 10t} = 1$	M1 A1
	$\Rightarrow 10t = 300 - 10t \Rightarrow t = 15$	M1 A1 (5)
(*d*)	Distance $= 300 \Rightarrow \lvert s - r \rvert^2 = 300^2$	M1
	$\Rightarrow (300 - 10t)^2 + (10t)^2 = 300^2$	M1 A1 ft
	$\Rightarrow 300^2 - 6000t + 100t^2 + 100t^2 = 300^2$	A1 ft
	$\Rightarrow 200t^2 = 6000t$	
	$t = 0$ or $30 \Rightarrow t = 30$	M1A1 (6)
		(15 marks)

Question Number	Scheme	Marks
1.	$0.5\mathbf{v} - 0.5\,(-20\mathbf{i}) = 15\mathbf{i} + 10\mathbf{j}$ $\Rightarrow \mathbf{v} = 10\mathbf{i} + 20\mathbf{j}$ $\therefore$ Speed $= \sqrt{(10^2 + 20^2)} \approx 22.4$ m s^{-1}	M1 A1 A1 M1 A1 ft (5) **(5 marks)**
2.	$F \times 0.02, = \frac{1}{2} \times 0.006\,(400^2 - 250^2)$ $F \approx 14600$ N	M1 A1, M1 A1 A1 ft (5) **(5 marks)**
3. (a)	$\mathbf{u} = (3t^2 - 3)\mathbf{i} + 8t\mathbf{j}$	M1 A1 (2)
(b)	$//^{e}\ \mathbf{i} + \mathbf{j} \Rightarrow 3t^2 - 3 = 8t$ $3t^2 - 8t - 3 = 0$ $(3t + 1)(t - 3) = 0$ $t = -\frac{1}{3}, 3 \quad t = 3$	M1 A1 ft M1 A1 A1 ft (5) **(7 marks)**
4.	$R(\uparrow)\ R = mg + 3mg = 4mg$ $R(\rightarrow)\ S = F$ M(A) $mg.a \sin\alpha + 3\,mg.\,2a \sin a = S.2a\cos\alpha$ $\rightarrow S = \frac{7}{2}\,mg\tan\alpha$  $\therefore F = S = \frac{7}{2}\,mg\tan\alpha,\ R = 4mg$ $F \le \frac{1}{4}R \Rightarrow \frac{7}{2}mg\tan\alpha \le mg \Rightarrow \tan\alpha \le \frac{2}{7}$	M1 A1 B1 M1 A1 A1 ft M1 M1 A1 (9) **(9 marks)**

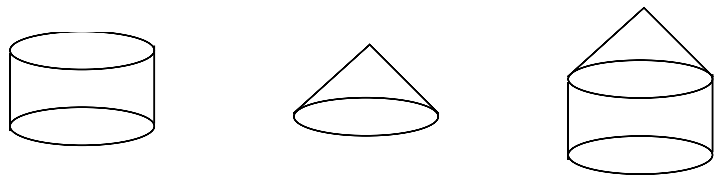
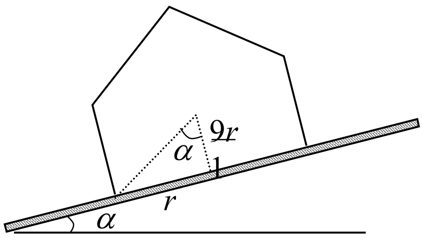
Question Number	Scheme	Marks
5. (a)	$F = 2000 + 4800g.\frac{1}{20}, = 4352$ N	M1 A1, A1
	P = 12 × 4652 W ≈ 52.2 kW	M1 A1 ft (5)
		
(b)	$4800a = 4352 - 2000$	M1 A1 ft
	$a = 0.49$ m s^{-2}	A1 (3)
(c)	Max speed $\frac{52224}{V} = 2000$	M1 A1
	$V \approx 26.1$ ms^{-1}	A1 (3)
		(11 marks)
6. (a)	Initial vertical speed = "$u \sin \alpha$" = $25\frac{5}{13}$ ms^{-1}	B1
	"$v^2 = u^2 + 2as$" $100 = 2gh$	M1
	$h = \frac{100}{2g} \approx 5.1$ m	A1
	∴ Ht + 5.1 + 0.8 = 5.9 m	A1 ft (4)
(b)	↔ Horizontal speed = "$u \cos \alpha$" = 24 ms^{-1}	B1
	Time to window $36 = 24t \Rightarrow t = 1.5$s	M1 A1
	$h = 0.8 + 10 \times 1.5 - \frac{1}{2} \times 9.8 \times 1.5^2$	M1 A1 A1 ft
	≈ 4.8 m	A1 (7)
(c)	One of, e.g., air resistance; spin of ball; variation in g; wind.	B1 (1)
		(12 marks)

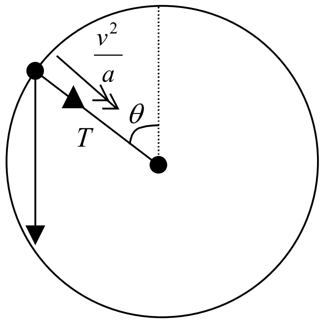
Question Number	Scheme	Marks
7. (a)	Ht of $\Delta = \sqrt{(15^2 - 9^2)}$	M1
	$= 12$ cm	A1
		
	Area 324 108 432	M1 A1
	Distance of CM from *AE* 9 $18 + \frac{1}{3}.12 = 22$ $\bar{x}$	B1 B1 ft
	$9.324 + 22.108 = 432\bar{x}$	M1 A1
	$\bar{x} = 12.25$ cm	A1 (9)
(b)	Distance of *G* from *BD* = 9 cm	B1
	$\tan\theta = \dfrac{18 - 12.25}{9}$	M1 A1
	$\theta = 32.6°$	A1 (4)
		(13 marks)

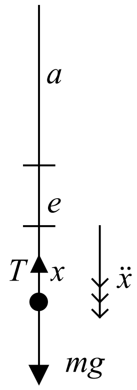
Question Number	Scheme	Marks
8.		
(a)	$3mu - 2mu = 2mw - mv$	M1 A1
	$4eu \quad = w + v$	M1 A1
	Solve $w = \frac{1}{3}(1 + 4e)u$	M1 A1 (6)
(b)	$v = \frac{1}{3}(8e - 1)u$	M1 A1
	$v > 0 \Rightarrow e > \frac{1}{8}$	A1 (3)
(c)	rebound speed of $B = \frac{1}{6}(1 + 4e)u$	B1
	2^{nd} collision $\Rightarrow \frac{1}{6}(1 + 4e)u > \frac{1}{3}(8e - 1)u$	M1
	$1 + 4e > 16e - 2$	
	$3 > 12e$	
	$e < \frac{1}{4}$	M1 A1 (4)
		(13 marks)

Question Number	Scheme	Marks
1.		
	$R(\uparrow)$ $N\cos\alpha = mg$	M1 A1
	$R(\rightarrow)$ $N\sin\alpha = \frac{m.14^2}{100}$	M1 A1
	$\therefore \tan\alpha = \frac{14^2}{100 \times 9.8} = 0.2$	M1 A1 ft
	$\alpha \approx 11.3°$	A1 (7)
		(7 marks)
2. (*a*)	$l^2 = 36^2 + 15^2$	M1
	$\Rightarrow l = 39$, ext = 9 cm	A1
	$T = \frac{\lambda \times 0.09}{0.3}$	B1
	$2T\sin\theta = mg \Rightarrow \frac{2\lambda \times 0.09}{0.3} \times \frac{15}{39} = 2 \times 9.8$	M1 A1
		
	$\lambda \approx 84.9$	A1 (6)
(*b*)	By taking P as single <u>point</u> from which to measure all distances	B1 (1)
		(7 marks)

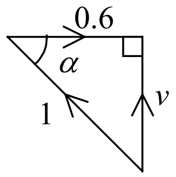
Question Number	Scheme	Marks
3.	$0.5\ddot{x} = -\frac{2}{x^2}$	M1
	$v\frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{4}{x^2}$	M1
	$\int v\,\mathrm{d}v = -\int \frac{4}{x^2}\,\mathrm{d}x$	M1
	$\left[\frac{1}{2}v^2\right]_3^{\frac{3}{2}} = \left[\frac{4}{x}\right]_1^d$	M1 A1
	(limits or 'C')	A1
	$\frac{9}{8} - \frac{9}{2} = 4\left(\frac{1}{d} - 1\right) \Rightarrow d = \frac{32}{5} = 6.4\text{ m}$	M1 A1 (8)
		(8 marks)
4. (*a*)	Elastic energy gained $= \frac{\lambda x^2}{2l}$	M1
	$\therefore \frac{\lambda.6^2}{2\times 12} = \text{PE lost} = 75\times 9.8\times 18$	M1 A1
	$\rightarrow \lambda = 8820\text{ N}$	M1 A1 ft (5)
(*b*)	At 2 m off ground $\frac{1}{2}\times 75\times v^2 = 75\times 9.8\times 17 - \frac{1}{2}\times\frac{8820\times 5^2}{12}$	M1 A1 A1ft
	$\rightarrow v^2 = 88.2$	
	$v \approx 9.39\text{ ms}^{-1}$	M1 A1 (5)
		(10 marks)

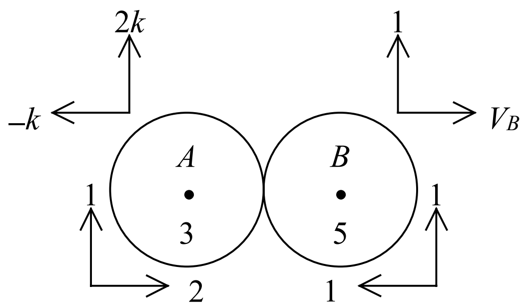
Question Number	Scheme	Marks
5.		
(*a*)		
	Vol. $\quad \pi r^3 \quad \frac{1}{3}\pi r^2 h \quad \pi r^3 + \frac{1}{3}\pi r^2 h$	M1 A1
	Dist of CM $\quad \frac{r}{2} \quad r + \frac{h}{4} \quad \bar{x}$	B1 B1
	$\frac{\pi r^4}{2} + \frac{1}{3}\pi r^2 h\left(r + \frac{h}{4}\right) = \left(\pi r^3 + \frac{1}{3}\pi r^2 h\right)\bar{x}$	M1 A1 A1ft
	$\rightarrow \bar{x} = \underline{\frac{6r^2 + 4hr + h^2}{4(3r+h)}}$	A1 (8)
(*b*)	 $h = 2r \Rightarrow \bar{x} = \frac{18r}{20} = \frac{9r}{10}$	M1 A1
	$\therefore \tan\alpha = \frac{r}{9r/10} = \frac{10}{9}$	M1 A1 ft
	$\alpha \approx 48^\circ$	A1 (5)
		(13 marks)

Question Number	Scheme	Marks
6.		
(*a*)	$R(\searrow)$ $T + mg\cos\theta = \dfrac{mv^2}{a}$	M1 A1
	Energy $\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mga(1+\cos\theta)$	M1 A1 A1
	$u^2 = 3ga \rightarrow v^2 = ga\ (1 - 2\cos\theta)$	
	$\therefore T = -mg\cos\theta + \dfrac{mv^2}{a} = mg(-3\cos\theta + 1)$	M1 A1
	$T = 0 \Rightarrow \cos\theta = \dfrac{1}{3}$	M1 A1 (9)
(*b*)	$v^2 = \dfrac{ga}{3}$	B1
	$\sin^2\theta = 1 - \left(-\dfrac{1}{3}\right)^2 = \dfrac{8}{9}$	M1 A1
	Ht $= \dfrac{v^2\sin^2\theta}{2g} = \dfrac{ga}{3}.\dfrac{8}{9}.\dfrac{1}{2g} = \dfrac{4a}{27}$	M1 M1 A1
		(6)
		(15 marks)

Question Number	Scheme	Marks
7.		
(*a*)	In equilibrium $\dfrac{6mge}{a} = mg \Rightarrow e = \dfrac{a}{6}$	M1 A1 (2)
(*b*)	$m\ddot{x} = -\dfrac{6mg(e+x)}{a} + mg$	M1 A1 A1
	$\rightarrow \ddot{x} = -\dfrac{6g}{a}x \Rightarrow$ SHM	M1 A1
	Period $= \left(\dfrac{2\pi}{\omega}\right) = 2\pi\sqrt{\dfrac{a}{6g}}$	A1 (6)
(*c*)	Greatest speed $= a\omega = \dfrac{a}{3}\sqrt{\dfrac{6g}{a}} = \dfrac{1}{3}\sqrt{6ga}$	M1 A1 (2)
(*d*)	$x = \dfrac{a}{3}\cos\omega t$	M1
	String slack $\Rightarrow x = -e \Rightarrow -\dfrac{a}{6} = \dfrac{a}{3}\cos\omega t$	M1 A1
	$\Rightarrow \omega t = \dfrac{2\pi}{3},\ t = \dfrac{2\pi}{3}\sqrt{\dfrac{a}{6g}}$	M1 A1 ft

		(5)
		(15 marks)

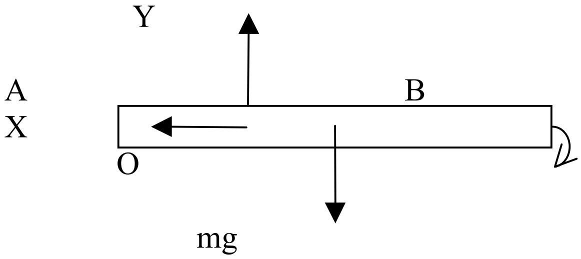
Question Number	Scheme	Marks
1.	$-4v = 2\dfrac{\mathrm{d}v}{\mathrm{d}t}$	M1 A1
	$-2\mathrm{d}t = \dfrac{\mathrm{d}v}{v}$	M1
	$-2t = \ln v$; $(-\ln 5)$	A1 ft; A1
	$v = 5\mathrm{e}^{-2t}$	A1 (6)
		(6 marks)
2. (*a*)	 (vector triangle)	M1
	$\cos\alpha = 0.6$	M1
	$\alpha = 53.1°$ upstream to bank	A1 (3)
(*b*)	$v = \sqrt{1^2 - 0.6^2}$	M1
	$= 0.8\ \mathrm{ms}^{-1}$	A1
	Time $= \dfrac{336}{0.8} = \underline{420\,\mathrm{s}}$	A1 ft (3)
		(6 marks)
3. (*a*)	$-\left(mg + mkv^2\right) = mv\dfrac{\mathrm{d}v}{\mathrm{d}s}$	M1 A1
	$\displaystyle\int_0^H \mathrm{d}s = \int_{\sqrt{\frac{g}{k}}}^{0} \frac{v\,\mathrm{d}v}{g + kv^2}$	M1 A1 A1
	$H = \dfrac{1}{2k}\Big[\ln(g + kv^2)\Big]_0^{\sqrt{\frac{g}{k}}}$	M1 A1
	$= \dfrac{1}{2k}\ln 2$	M1 A1 (9)
(*b*)	Spin, variation in g	B1 (1)
		(10 marks)

Question Number	Scheme	Marks
4. (*a*)	 $V_A = k\begin{pmatrix}-1\\2\end{pmatrix}$	M1 A1
	but $2k = 1 \Rightarrow k = \frac{1}{2}$	M1 A1 ft
	$\therefore V_A = \begin{pmatrix}-\frac{1}{2}\\1\end{pmatrix}$	A1 ft (5)
(*b*)	CLM: $(3\times 2) - (5\times 1) = \left(3\times -\frac{1}{2}\right) + 5V_B$	M1 A1 ft
	NIL: $V_B + \frac{1}{2} = e(2+1)$	M1 A1 ft
	Solving	M1
	$e = \frac{1}{3}$	A1 (6)
		(11 marks)
5. (*a*)	In equilibrium: $mg = 2\frac{mge}{l} \Rightarrow e = \frac{1}{2}l$	M1 A1
	$mg - T - 2m\omega\dot{x} = m\ddot{x}$	M1 A1 A1
	$mg - \frac{2mg}{l}\left(x + \frac{1}{2}l\right) - 2m\omega\dot{x} = \ddot{x}$	M1
	$\ddot{x} + 2\omega\dot{x} + 2\omega^2 x = 0$	A1 (7)
(*b*)	AE: $m^2 + 2\omega m + 2\omega^2 = 0$	M1
	$m = -\omega(1 \pm i)$	A1
	$\underline{x = e^{-\omega t}(A\cos\omega t + B\sin\omega t)}$ $\left(\omega = \sqrt{\frac{g}{l}}\right)$	M1 A1 ft (4)
(*c*)	Period $= 2\pi\sqrt{\frac{l}{g}}$	B1 (1)
		(12 marks)

Question Number	Scheme	Marks
6. (a)	$V_T \ \uparrow 6 \text{ ms}^{-1} \quad V_C \leftarrow 24 \text{ ms}^{-1}$	B1
	$V_{C-T} = V_C - V_T$	B1
	$\left\| {}_C V_T \right\| = \sqrt{6^2 + 24^2} = \underline{24.7 \text{ ms}^{-1}}$	M1 A1 ft
	Direction = arctan $\left(\frac{6}{24}\right)$ = 14.04°, bearing of 256°	M1 A1 ft (6)
(b)	$s = \sqrt{200^2 + 960^2} = 980.6$	M1 A1
	$\alpha = \arctan \frac{200}{960} = 11.77^\circ$	M1 A1
	$\beta = \theta - \alpha = 14.04 - 11.77 = 2.27^\circ$	M1 A1 ft
	$p = s \sin \beta = 38.8 \text{ m}$	M1 A1 (8)
		(14 marks)
7. (a)	$-mga \sin\theta; \frac{mg}{2a}(2a \sin\theta - a)^2$	B1; M1 A1
	$V = -mga \sin\theta + \frac{mg}{2a}(2a\sin\theta - a)^2 + c$	M1
	$= -mga \sin\theta + \frac{mg}{2a}\left(4a^2 \sin^2\theta - 4a \sin\theta + a^2\right) + c$	A1
	$= \underline{mga\left(2\sin^2\theta - 3\sin\theta\right) + \text{constant}}$	A1 (6)
(b)	$\frac{\mathrm{d}V}{\mathrm{d}\theta} = mga(4\sin\theta \times \cos\theta - 3\cos\theta)$	M1 A1 A1
	$= mga\cos\theta(4\sin\theta - 3) = 0$	M1
	$\theta = \arcsin\left(\frac{3}{4}\right)$	
	$= \underline{0.848^c}$	A1 (5)
(c)	$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = mga\,(4\cos 2\theta + 3\sin\theta)$	M1 A1
	$\theta = \arcsin\left(\frac{3}{4}\right) : V'' = mga\left(-\frac{4}{8} + \frac{9}{4}\right) = \frac{7}{4}mga \quad \therefore$ Stable	M1 A1 A1 ft (5)
		(16 marks)

Question Number		Scheme	Marks	
1.	(*a*)	$\overrightarrow{AB} = (\mathbf{i} - 5\mathbf{j})$ m	B1	
		$(14\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})\ (\mathbf{i} - 5\mathbf{j}) = 4$J	M1 A1	(3)
	(*b*)	$\frac{1}{2} \times 0.125 \times v^2 = 4$	M1	
		$v = 8$ ms^{-1}	A1 ft	(2)
				(5 marks)
2.	(*a*)	$\mathrm{I} = \int_{-a}^{a} \frac{m}{2a} x^2 \mathrm{d}x$	M1 A1	
		$= \frac{ma^2}{3}$	A1	(3)
	(*b*)	$I_x = I_y = 2\left(\frac{ma^2}{3} + ma^2\right) = \frac{8ma^2}{3}$	M1 A1	
		$\therefore$ by perpendicular axes, $I_z = \frac{16ma^2}{3}$	M1 A1 ft	(4)
				(7 marks)
3.	(a)	$\mathbf{F} = (3\mathbf{i} + 4\mathbf{j}) + (2\mathbf{i} - \mathbf{j} + \mathbf{k})$	M1	
		$= (5\mathbf{i} + 3\mathbf{j} + \mathbf{k})$ N	A1	(2)
	(b)	Moment of $\mathbf{F}_1, \mathbf{F}_2$ about 0 $= \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$	M1	
		$= \begin{pmatrix} 0 \\ 0 \\ 8 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ -3 \end{pmatrix}$	A1 A1	
		$\therefore \begin{pmatrix} 1 \\ -1 \\ 5 \end{pmatrix} + \mathbf{G} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \mathbf{G} = \begin{pmatrix} -1 \\ 1 \\ -5 \end{pmatrix}$ Nm $= (-\mathbf{i} + \mathbf{j} - 5\mathbf{k})$ Nm	M1 A1	(5)
				(7 marks)

Question Number		Scheme	Marks
4.	(*a*)	$I = \left(\frac{1}{2} \times 2ma^2 + 2ma^2\right) + m\left(a\sqrt{2}\right)^2$	M1 A1 A1
		$= 5\ ma^2$	A1
		$ma\sqrt{20ag} \quad = 5ma^2\ \omega$	M1 A1
		$\omega = \sqrt{\frac{4g}{5a}}$	A1 (7)
	(*b*)	PE Gain = 2 mga	A1
		KE Loss $= \frac{1}{2} \times 5ma^2 \times \frac{4g}{5a} = 2mga$	M1 A1 (3)
			(10 marks)
5.	(a)	$I_{AB} = \frac{1}{3} ma^2$	B1
		$I_A = 2 \times \frac{1}{3} ma^2$ (perpendicular axes)	M1 A1 (3)
	(b)	$M(A), mg\frac{a}{\sqrt{2}} \sin\theta = -\frac{2}{3} ma^2\ \ddot{\theta}$	M1 A1 A1
		$\ddot{\theta} \approx \frac{-3g}{2a\sqrt{2}}\theta$ for small θ, hence SHM	M1 A1 (5)
	(c)	$t = \frac{1}{4} \times$ period $= \frac{\pi}{2}\sqrt{\frac{2a\sqrt{2}}{3g}}$	M1 A1 (2)
		$\left(= \pi\sqrt{\frac{a\sqrt{2}}{6g}}\right)$	
			(10 marks)

Question Number	Scheme	Marks
6. (*a*)	 $I_0 = \frac{1}{12}m(AB)^2 + ma^2 = \frac{7}{3}ma^2$	M1 A1
	$mga = \frac{7ma^2}{3}\ddot{\theta}$	M1
	$\ddot{\theta} = \frac{3g}{7a}$	A1 (4)
(*b*)	$\frac{1}{2} \times \frac{7ma^2}{3}\dot{\theta}^2 = mga \Rightarrow a\dot{\theta}^2 = \frac{6g}{7}$	M1 A1 (2)
(*c*)	$R(\downarrow)$: $mg - Y = ma.\frac{3g}{7a} \Rightarrow Y = \frac{4mg}{7}$	M1 A1
	$R(\leftarrow)$: $X = ma\dot{\theta}^2 = \frac{6mg}{7}$	M1
	$R = \frac{mg}{7}\sqrt{4^2 + 6^2} = \frac{mg}{7}\sqrt{52}$	M1 A1 (5)
		(11 marks)

Question Number	Scheme	Marks
7. (*a*)	$m = \frac{4}{3}\pi r^3 \rho \quad (\rho \text{ constant})$	
	$\frac{\mathrm{d}m}{\mathrm{d}t} = \frac{4}{3}\pi\rho \times 3r^2 \frac{\mathrm{d}r}{\mathrm{d}t} = 4\pi\rho r^2 .kr = 3\,km$	M1 A1 (2)
(*b*)	$mg\delta t = (m + \delta m)(v + \delta v) - mv$	M1
	$mg = v\frac{\mathrm{d}m}{\mathrm{d}t} + m\frac{\mathrm{d}v}{\mathrm{d}t}$	A1
	$mg = v \times 3km + m\frac{\mathrm{d}v}{\mathrm{d}t}$	M1
	$g - 3kv = \frac{\mathrm{d}v}{\mathrm{d}t}$	A1 (4)
(*c*)	$\int \mathrm{d}t = \int \frac{\mathrm{d}v}{g - 3kv}$	M1
	$t = -\frac{1}{3k}\ln(g - 3kv)(+c)$	A1
	$t = 0, v = u:\ c = \frac{1}{3k}\ln(g - 3ku)$	A1
	$\frac{g - 3ku}{g - 3kv} = e^{3kt}$ (or equivalent)	M1
	$v = \frac{g}{3k} - \left(\frac{g}{3k} - u\right)e^{-3kt}$	A1 (5)
(d)	As $t \to \infty$, $\mathrm{e}^{-3kt} \to 0$	
	$v \to \frac{g}{3k}$	B1 (1)
		(12 marks)

Question Number	Scheme	Marks
8. (*a*)	$m^2 + 9 = 0 \Rightarrow m = \pm 3\mathbf{i}$	M1
	$r = A \sin 3t + B \cos 3t$	A1
	Let $r = p \sin t\mathbf{i}$ $\dot{r} = p \cos t\mathbf{i}$ $\ddot{r} = -p \sin t\mathbf{i}$	M1 A1
	$-p \sin t\mathbf{i} + 9p \sin t\mathbf{i} = 8 \sin t\mathbf{i}$	M1
	$\Rightarrow p = 1$	A1
	$r = A \sin 3t + B \cos 3t + \sin t\mathbf{i}$	M1
	$t = 0$: $0 = B$	A1
	$\dot{r} = 3A \cos 3t + \cos 3t + \cos t\mathbf{i}$	M1
	$t = 0$: $\mathbf{i} + 3\mathbf{j} = 3A + \mathbf{i} \Rightarrow A = \mathbf{j}$	A1
	$\therefore r = \sin t\mathbf{i} + \sin 3t\mathbf{j}$	A1 (11)
(*b*)	$\sin t = \sin 3t = 0$	M1
	$\Rightarrow t = \pi$	A1 (2)
		(13 marks)