GCE Examinations

Mechanics
Module M3

Advanced Subsidiary / Advanced Level

Paper D

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

When a numerical value of g is required, use $g = 9.8 \text{ m s}^{-2}$.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

1. The mechanism for releasing the ball on a pinball machine contains a light elastic spring of natural length 15 cm and modulus of elasticity λ.

The spring is held compressed to a length of 9 cm by a force of 4.5 N.

(a) Find λ. **(3 marks)**

(b) Find the work done in compressing the spring from a length of 9 cm to a length of 5 cm.

(4 marks)

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2. A small bead P is threaded onto a smooth circular wire of radius 0.8 m and centre O which is fixed in a vertical plane.

The bead is projected from the point vertically below O with speed u m s^{-1} and moves in complete circles about O.

(a) Suggest a suitable model for the bead. **(1 mark)**

(b) Given that the minimum speed of P is 60% of its maximum speed, use the principle of conservation of energy to show that $u = 7$.

(6 marks)

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3. At time t seconds the acceleration, a m s^{-2}, of a particle is given by

$$a = \frac{4}{(1+t)^3} \, .$$

When $t = 0$, the particle has velocity 1 m s^{-1} and displacement 3 m from a fixed origin O.

(a) Find an expression for the velocity of the particle in terms of t. **(4 marks)**

(b) Show that when $t = 3$ the particle is 10.5 m from O. **(5 marks)**

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4. A particle of mass 0.5 kg is moving on a straight line with simple harmonic motion.

At time $t = 0$ the particle is instantaneously at rest at the point A. It next comes instantaneously to rest 3 seconds later at the point B where $AB = 4$ m.

(a) For the motion of the particle write down

   (i) the period,

   (ii) the amplitude. **(2 marks)**

(b) Find the maximum kinetic energy of the particle in terms of π. **(4 marks)**

The point C lies on AB at a distance of 1.2 m from B.

(c) Find the time it takes the particle to travel directly from A to C, giving your answer in seconds correct to 2 decimal places.

   (4 marks)

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5. When a particle of mass M is at a distance of x metres from the centre of the moon, the gravitational force, F N, acting on it and directed towards the centre of the moon is given by

$$F = \frac{(4.90 \times 10^{12})M}{x^2} .$$

A rocket is projected vertically into space from a point on the surface of the moon with initial speed u m s^{-1}. Given that the radius of the moon is (1.74×10^6) m,

(a) show that the speed of the rocket, v m s^{-1}, when it is x metres from the centre of the moon is given by

$$v^2 = u^2 + \frac{a}{x} - b,$$

where a and b are constants which should be found correct to 3 significant figures.

   (7 marks)

(b) Find, correct to 2 significant figures, the minimum value of u needed for the rocket to escape the moon's gravitational attraction.
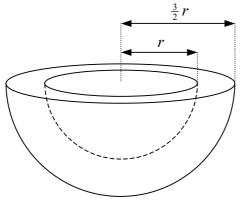
   (4 marks)

6.

$\tfrac{3}{2}r$

r

Fig. 1

Figure 1 shows a bowl formed by removing from a solid hemisphere of radius $\tfrac{3}{2}r$ a smaller hemisphere of radius r having the same axis of symmetry and the same plane face.

(a) Show that the centre of mass of the bowl is a distance of $\tfrac{195}{304}r$ from its plane face.

(7 marks)

The bowl has mass M and is placed with its curved surface on a smooth horizontal plane. A stud of mass $\tfrac{1}{2}M$ is attached to the outer rim of the bowl.
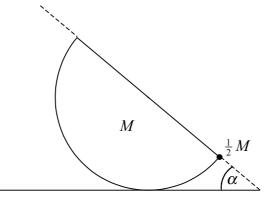

M

$\tfrac{1}{2}M$

α

Fig. 2

When the bowl is in equilibrium its plane surface is inclined at an angle α to the horizontal as shown in Figure 2.

(b) Find $\tan \alpha$. **(6 marks)**

7. A cyclist is travelling round a circular bend of radius 25 m on a track which is banked at an angle of 35° to the horizontal.

In a model of the situation, the cyclist and her bicycle are represented by a particle of mass 60 kg and air resistance and friction are ignored.

Using this model and assuming that the cyclist is not slipping,

(a)   find, correct to 3 significant figures, the speed at which she is travelling.   **(5 marks)**

In tests it is found that the cyclist must travel at a minimum speed of 10 m s^{-1} to prevent the bicycle from slipping down the slope. A more refined model is now used with a coefficient of friction between the bicycle and the track of μ.

Using this model,

(b)   show that $\mu = 0.227$, correct to 3 significant figures,   **(8 marks)**

(c)   find, correct to 2 significant figures, the maximum speed at which the cyclist can travel without slipping up the slope.

(5 marks)

END