GCE Examinations

Advanced Subsidiary / Advanced Level

# Mechanics
# Module M3

Paper D

# MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

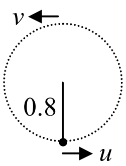
Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

*Written by Shaun Armstrong & Chris Huffer*

# M3 Paper D – Marking Guide

| | | | | |
|---|---|---|---|---|
| **1.** | *(a)* | $T = \frac{\lambda x}{l}$ $\therefore$ $4.5 = \frac{0.06\lambda}{0.15}$ | M1 A1 | |
| | | giving $\lambda = 11.25$ N | A1 | |
| | *(b)* | work done = change in EPE = $\frac{\lambda}{2l}(x_2{}^2 - x_1{}^2)$ | M1 A1 | |
| | | $= \frac{11.25}{2\times 0.15}(0.1^2 - 0.06^2) = 0.24$ J | M1 A1 | **(7)** |
| **2.** | *(a)* | particle | B1 | |
| | *(b)* | minimum speed when bead is at highest point | B1 | |
| | |   con. of ME: $\frac{1}{2}m(u^2 - v^2) = mg \times 1.6$ | M1 A1 | |
| | | $v = \frac{3}{5}u$ | M1 | |
| | | $\therefore u^2 - \frac{9}{25}u^2 = 3.2g$ | M1 | |
| | | $u^2 = 3.2 \times 9.8 \div \frac{16}{25} = 49$ so $u = 7$ | A1 | **(7)** |
| **3.** | *(a)* | $v = \int \frac{4}{(1+t)^3}\, dt$ $\therefore$ $v = \frac{-2}{(1+t)^2} + c$ | M1 A1 | |
| | | $t = 0, v = 1$ $\therefore c = 3$ | M1 | |
| | | giving $v = [3 - \frac{2}{(1+t)^2}]$ ms$^{-1}$ | A1 | |
| | *(b)* | $x = \int 3 - \frac{2}{(1+t)^2}\, dt$ $\therefore$ $x = 3t + \frac{2}{(1+t)} + d$ | M1 A1 | |
| | | $t = 0, x = 3$ $\therefore d = 1$ | M1 | |
| | | giving $x = 3t + \frac{2}{(1+t)} + 1$ | | |
| | | $t = 3,$ $x = 9 + \frac{1}{2} + 1 = 10.5$ m | M1 A1 | **(9)** |
| **4.** | *(a)* | (i) period $= 2 \times 3 = 6$ s | B1 | |
| | | (ii) amplitude $= \frac{1}{2} \times 4 = 2$ m | B1 | |
| | *(b)* | period $= 6 = \frac{2\pi}{\omega}$ $\therefore$ $\omega = \frac{\pi}{3}$ | M1 | |
| | | $v_{max} = a\omega = 2 \times \frac{\pi}{3} = \frac{2\pi}{3}$ | A1 | |
| | | $KE_{max} = \frac{1}{2}mv_{max}{}^2 = \frac{1}{2} \times \frac{1}{2} \times (\frac{2\pi}{3})^2 = \frac{1}{9}\pi^2$ J | M1 A1 | |
| | *(c)* | $x = a\cos\omega t$ $\therefore$ $^{-}0.8 = 2\cos\omega t$ | M1 A1 | |
| | | $\cos\omega t = {}^{-}0.4$ $\therefore$ $\omega t = 1.9823$ $\therefore$ $t = 1.9823 \div \frac{\pi}{3} = 1.89$ s (2dp) | M1 A1 | **(10)** |
| **5.** | *(a)* | $F = ma = Mv\frac{dv}{dx} = -\frac{4.90\times 10^{12} \times M}{x^2}$ | M1 A1 | |
| | | $\int v\, dv = {}^{-}(4.90 \times 10^{12}) \int x^{-2}\, dx$ | M1 | |
| | | $\frac{1}{2}v^2 = (4.90 \times 10^{12})x^{-1} + c$ | A1 | |
| | | $x = (1.74 \times 10^6),\ v = u$ $\therefore$ $c = \frac{1}{2}u^2 - \frac{4.90\times 10^{12}}{1.74\times 10^6}$ | M1 A1 | |
| | | so $v^2 = u^2 + (9.80 \times 10^{12})x^{-1} - (5.63 \times 10^6)$ | A1 | |
| | *(b)* | we require $v > 0$ as $x \to \infty$ | M1 | |
| | | $x^{-1} \to 0$ $\therefore$ $u^2 - (5.63 \times 10^6) > 0$ | M1 A1 | |
| | | giving $u_{min} = 2400$ ms$^{-1}$ (2sf) | A1 | **(11)** |

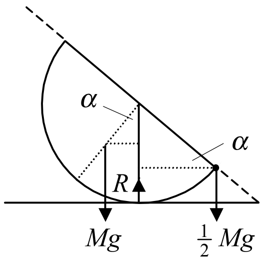
**6.** *(a)*

| portion | mass | $y$ | $my$ |
|---|---|---|---|
| large hemisphere | $\frac{1}{2}\times\rho\times\frac{4}{3}\pi(\frac{3}{2}r)^3 = \frac{9}{4}\rho\pi r^3$ | $\frac{3}{8}\times\frac{3}{2}r = \frac{9}{16}r$ | $\frac{81}{64}\rho\pi r^4$ |
| small hemisphere | $\frac{1}{2}\times\rho\times\frac{4}{3}\pi r^3 = \frac{2}{3}\rho\pi r^3$ | $\frac{3}{8}r$ | $\frac{1}{4}\rho\pi r^4$ |
| bowl | $\frac{19}{12}\rho\pi r^3$ | $\bar{y}$ | $\frac{65}{64}\rho\pi r^4$ |

$\rho$ = mass per unit volume  $y$ coords. taken vert. from plane face — M2 A3

$\frac{19}{12}\rho\pi r^3 \times \bar{y} = \frac{65}{64}\rho\pi r^4 \quad \therefore \quad \bar{y} = \frac{65}{64}r \div \frac{19}{12} = \frac{195}{304}r$ — M1 A1

*(b)*



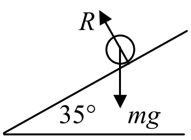

mom. about vert. through pt of contact

$Mg \times \frac{195}{304}r\sin\alpha = \frac{1}{2}Mg \times \frac{3}{2}r\cos\alpha$ — M2 A2

$\therefore \tan\alpha = \frac{3}{4} \div \frac{195}{304} = \frac{76}{65}$ — M1 A1 **(13)**

---

**7.** *(a)*



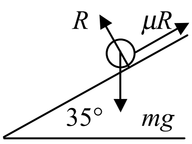

resolve ↑: $R\cos 35 - mg = 0$, $R = \frac{mg}{\cos 35}$ — M1

resolve ←: $R\sin 35 = \frac{mv^2}{r} = \frac{mv^2}{25}$ — M1 A1

combining, $v^2 = 25g\tan 35$ — M1

giving $v = 13.1\text{ ms}^{-1}$ (3sf) — A1

*(b)*




resolve ↑: $R\cos 35 + \mu R\sin 35 - mg = 0$ — M1 A1

$R = \frac{mg}{\cos 35 + \mu\sin 35}$

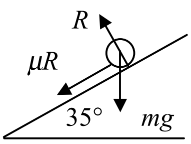
resolve ←: $R\sin 35 - \mu R\cos 35 = \frac{mv^2}{r} = \frac{100m}{25}$ — M1 A1

combining, $\frac{mg(\sin 35 - \mu\cos 35)}{\cos 35 + \mu\sin 35} = 4m$ — M1 A1

giving $g(\sin 35 - \mu\cos 35) = 4(\cos 35 + \mu\sin 35)$ — M1

$\therefore \mu = \frac{g\sin 35 - 4\cos 35}{4\sin 35 + g\cos 35} = 0.227$ (3sf) — A1

*(c)*




resolve ↑: $R\cos 35 - \mu R\sin 35 - mg = 0$ — M1

$R = \frac{mg}{\cos 35 - \mu\sin 35}$

resolve ←: $R\sin 35 + \mu R\cos 35 = \frac{mv^2}{r} = \frac{mv^2}{25}$ — M1

combining, $\frac{mg(\sin 35 + \mu\cos 35)}{\cos 35 - \mu\sin 35} = \frac{mv^2}{25}$ — M1

giving $v^2 = \frac{25g(\sin 35 + \mu\cos 35)}{\cos 35 - \mu\sin 35}$ — M1

$\therefore v = 16\text{ ms}^{-1}$ (2sf) — A1 **(18)**

---

Total **(75)**

# Performance Record – M3 Paper D

| Question no. | **1** | **2** | **3** | **4** | **5** | **6** | **7** | Total |
|---|---|---|---|---|---|---|---|---|
| Topic(s) | elastic spring, EPE | motion in a vertical circle | variable accel. | SHM | variable force | centre of mass, equilm. | circular motion, banked track | |
| Marks | 7 | 7 | 9 | 10 | 11 | 13 | 18 | 75 |
| Student | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |