

GCE Examinations

Advanced Subsidiary / Advanced Level

Mechanics
Module M3

Paper E

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



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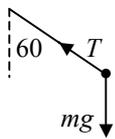
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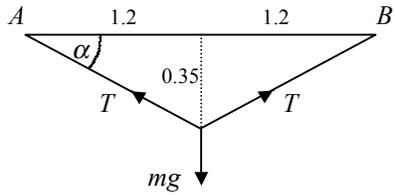
M3 Paper E – Marking Guide

1. (a) $\mathbf{a} = \frac{d}{dt}(\mathbf{v}) = (2e^{2t}\mathbf{i} + 2\mathbf{j}) \text{ cms}^{-2}$ M1 A1
- (b) $2e^{2t}\mathbf{i} + 2\mathbf{j} = k(4\mathbf{i} + \mathbf{j})$, comparing \mathbf{j} components, $k = 2$ M1 A1
 $\therefore 2e^{2t} = 8$, $t = \frac{1}{2} \ln 4 = 0.69 \text{ s}$ (2sf) M1 A1
- (c) e.g. predicts \mathbf{v} , \mathbf{a} increasing to very large values B1 (7)

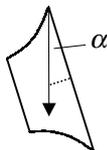
2. (a) work done $= \int_0^1 5 + 4e^{-x} dx = [5x - 4e^{-x}]_0^1$ M1 A1
 $= (5 - 4e^{-1}) - (0 - 4) = 9 - 4e^{-1} = 7.53 \text{ J}$ (3sf) M1 A1
- (b) work done = change in KE M1
 $\therefore 9 - 4e^{-1} = \frac{1}{2} \times 0.4 \times (v^2 - 2^2)$ M1 A1
giving $v = 6.45 \text{ ms}^{-1}$ (3sf) A1 (8)

3. (a)  resolve \uparrow : $T \cos 60^\circ - mg = 0$ M1
 $\therefore \frac{1}{2} T = 0.1 \times 9.8$ A1
giving $T = 1.96 \text{ N}$ A1
- (b) resolve \leftarrow : $T \sin 60^\circ = mr\omega^2$ M1 A1
let length of string $= l \therefore r = l \sin 60^\circ$ so $1.96 = 0.1 \times l \times 4^2$ M1 A1
giving $l = 1.225 \text{ m}$ A1 (8)

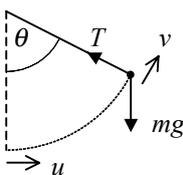
4. (a) $\ddot{x} = -\omega^2 x \therefore 0.12 = |-\omega^2 \times 0.03|$ giving $\omega = 2$ M1 A1
period $= \frac{2\pi}{\omega} = \pi$ seconds M1 A1
- (b) $v^2 = \omega^2(a^2 - x^2) \therefore 0.08^2 = 2^2(a^2 - 0.03^2)$ M1 A1
giving $a = 0.05 \text{ m}$ or 5 cm A1
- (c) $v_{\max} = \omega a = 2 \times 0.05 = 0.1 \text{ ms}^{-1}$ or 10 cms^{-1} M1 A1 (9)

5. (a) by symmetry B1
- (b)  by Pythag. distance of object from A $= 1.25$ M1
 $T = \frac{\lambda x}{l} = \frac{280 \times 0.25}{1} = 70$ M1 A1
resolve \uparrow : $2T \sin \alpha - mg = 0$ M1 A1
 $\sin \alpha = \frac{7}{25} \therefore 2 \times 70 \times \frac{7}{25} = m \times 9.8$ M1
giving $m = 4 \text{ kg}$ A1
- (c) $\text{EPE} = 2 \times \frac{\lambda x^2}{2l} = 2 \times \frac{280 \times 0.25^2}{2 \times 1} = 17.5 \text{ J}$ M1 A1 (10)

6. (a) $\bar{x} \times \int_0^1 \rho \pi y^2 dx = \int_0^1 \rho \pi y^2 x dx \quad \therefore \bar{x} \times \int_0^1 y^2 dx = \int_0^1 y^2 x dx$ B1
 $\int_0^1 y^2 dx = \int_0^1 (x^2 + 1)^2 dx = \int_0^1 (x^4 + 2x^2 + 1) dx$ M1 A1
 $= [\frac{1}{5}x^5 + \frac{2}{3}x^3 + x]_0^1 = \frac{28}{15}$ M1 A1
 $\int_0^1 y^2 x dx = \int_0^1 x(x^2 + 1)^2 dx = \int_0^1 (x^5 + 2x^3 + x) dx$ M1
 $= [\frac{1}{6}x^6 + \frac{1}{2}x^4 + \frac{1}{2}x^2]_0^1 = \frac{7}{6}$ M1 A1
 $\therefore \bar{x} = \frac{7}{6} \div \frac{28}{15} = \frac{5}{8}, \bar{y} = 0$ by symmetry \therefore coords are $(\frac{5}{8}, 0)$ M1 A1

(b)  $\tan \alpha = \frac{1 - \frac{5}{8}}{\frac{5}{8}} = \frac{3}{5}$ M1 A1
 $\therefore \alpha = 11^\circ$ (nearest degree) A1 (13)

7. (a) full circles if $T \geq 0$ at max height B1
 resolve \downarrow : $T + mg = \frac{mv^2}{r} \quad \therefore \frac{mv^2}{0.6} \geq mg$ M1 A1
 con. of ME: $\frac{1}{2} m(u^2 - v^2) = mg \times 1.2$ M1 A1
 $\therefore v^2 = u^2 - 2.4g$ M1
 combining, $\frac{u^2 - 2.4g}{0.6} \geq g$ M1
 giving $u^2 \geq 3g \quad \therefore u \geq \sqrt{3g}$ A1

(b)  resolve \curvearrowright : $T - mg \cos \theta = \frac{mv^2}{r}$ M1
 when slack, $T = 0 \quad \therefore v^2 = -0.6g \cos \theta$ A1
 con. of ME: $\frac{1}{2} m(u^2 - v^2) = 0.6mg(1 - \cos \theta)$ M1 A1
 $\therefore v^2 = 25 - 1.2g(1 - \cos \theta)$ M1
 combining, $-0.6g \cos \theta = 25 - 1.2g(1 - \cos \theta)$ M1
 giving $\cos \theta = \frac{1.2g - 25}{1.8g} \quad \therefore \theta = 139^\circ$ (nearest deg) A1

(c) $v^2 = -0.6g(\frac{1.2g - 25}{1.8g})$ giving $v^2 = 4.413$ M1
 vertically: $u = \sqrt{4.413} \sin 41^\circ, a = -g, v = 0$ (for greatest height) M1
 using $v^2 = u^2 + 2as, 0 = 4.413 \sin^2 41^\circ - 2gs$ M1 A1
 giving $s = 0.098 \text{ m} = 10 \text{ cm}$ (nearest cm) A1 (20)

Total (75)

