GCE Examinations

Advanced Subsidiary / Advanced Level

Mechanics
Module M3

Paper C

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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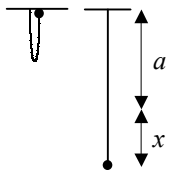
M3 Paper C – Marking Guide

1.



EPE = $\frac{\lambda x^2}{2l} = \frac{4mgx^2}{2a}$ M1 A1

con. of ME: $mg(a+x) = \frac{4mgx^2}{2a}$ M1 A1

$\therefore\ a(a+x) = 2x^2$ giving $2x^2 - ax - a^2 = 0$ A1

$(2x+a)(x-a) = 0\ \ \therefore x = -\frac{1}{2}a$ or a M1

$x > 0\ \ \therefore\ x = a$ so $AB = 2a$ A1 **(7)**

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2. *(a)* $\mathbf{a} = \frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{v}) = (\frac{1}{t+1}\mathbf{i} + 2\mathrm{e}^{-2t}\mathbf{j})$ ms^{-2} M1 A2

(b) $t = 1,\ \mathbf{a} = \frac{1}{2}\mathbf{i} + 2\mathrm{e}^{-2}\mathbf{j}$ M1

$|\mathbf{a}| = \sqrt{(\frac{1}{4} + 4\mathrm{e}^{-4})} = 0.5686$ M1 A1

$F = ma = 0.25 \times 0.5686 = 0.142$ N (3sf) A1 **(7)**

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3. *(a)* $\omega = \frac{45}{60} \times 2\pi = \frac{3}{2}\pi$ M1

$v = \omega r = \frac{3}{2}\pi \times 0.1 = \frac{3}{20}\pi$ or 0.47 ms^{-1} (2sf) A1

(b) resolve ↑: $R - mg = 0\ \ \therefore R = 0.005 \times 9.8 = 0.049$ N M1 A1

resolve ←: $F = ma = mr\omega^2 = 0.005 \times 0.1 \times (\frac{3}{2}\pi)^2 = 0.011$ N (2sf) M1 A1

$\therefore$ horiz. and vert. components are 0.011 N and 0.049 N respectively

(c) limiting friction $\therefore F = \mu R$ M1

$0.01110 = 0.049\mu\ \ \therefore \mu = \frac{0.01110}{0.049} = 0.23$ (2sf) M1 A1 **(9)**

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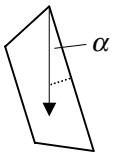
4. *(a)*

portion	mass	y	my
large cone	$\rho\frac{1}{3}\pi(3r)^2 3h = 9\rho\pi r^2 h$	$\frac{1}{4} \times 3h = \frac{3}{4}h$	$\frac{27}{4}\rho\pi r^2 h^2$
small cone	$\rho\frac{1}{3}\pi(2r)^2 2h = \frac{8}{3}\rho\pi r^2 h$	$h + \frac{1}{4} \times 2h = \frac{3}{2}h$	$4\rho\pi r^2 h^2$
frustrum	$\frac{19}{3}\rho\pi r^2 h$	$\bar{y}$	$\frac{11}{4}\rho\pi r^2 h^2$

ρ = mass per unit volume y coords. taken vert. from base M2 A3

$\frac{19}{3}\rho\pi r^2 h \times \bar{y} = \frac{11}{4}\rho\pi r^2 h^2\ \ \therefore\ \bar{y} = \frac{11}{4}h \div \frac{19}{3} = \frac{33}{76}h$ M1 A1

(b)



$\tan\alpha = \frac{\frac{33}{76} \times 2r}{3r} = \frac{11}{38}$ M2 A1

$\therefore\ \alpha = 16°$ (nearest degree) A1 **(11)**

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5. *(a)* $F = ma = 0.8\,v\dfrac{dv}{dx} = -\frac{k}{x^2}$ M1

$\therefore \int 4v\ dv = \int -\frac{5k}{x^2}\ dx$ M1

giving $2v^2 = \frac{5k}{x} + c$ A1

$x = 2, v = 5\ \therefore 50 = \frac{5k}{2} + c$ M1 A1

$x = 4, v = 3\ \therefore 18 = \frac{5k}{4} + c$ M1

solve simul. $32 = k(\frac{5}{2} - \frac{5}{4}) = \frac{5}{4}k$ M1

$\therefore k = \frac{32\times4}{5} = \frac{128}{5}$ A1

(b) $c = 50 - \frac{5k}{2} = -14\ \therefore v^2 = \frac{64}{x} - 7$ M1 A1

rest when $v = 0\ \therefore \frac{64}{x} = 7$ so $x = \frac{64}{7}$ m M1 A1 **(12)**

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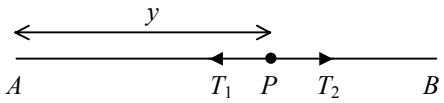
6. *(a)*



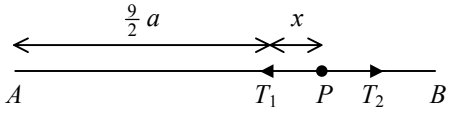
$T_1 = \frac{\lambda x}{l} = \frac{\lambda(y-3a)}{3a}$ M1 A1

$T_2 = \frac{2\lambda(5a-y)}{2a}$ A1

eqm. $\therefore T_1 = T_2,\ \frac{\lambda(y-3a)}{3a} = \frac{2\lambda(5a-y)}{2a}$ M1

giving $y - 3a = 3(5a - y)\ \therefore y = \frac{9}{2}a$ A1

(b)



$m\ddot{x} = T_2 - T_1 = \frac{2\lambda(\frac{1}{2}a-x)}{2a} - \frac{\lambda(\frac{3}{2}a+x)}{3a} = \frac{\lambda}{3a}[(\frac{3}{2}a - 3x) - (\frac{3}{2}a + x)]$ M2 A2

giving $\ddot{x} = -\frac{4\lambda}{3ma}x\ \therefore$ SHM with $\omega^2 = \frac{4\lambda}{3ma},\ \omega = 2\sqrt{\frac{\lambda}{3ma}}$ M1 A2

period $= \frac{2\pi}{\omega} = \pi\sqrt{\frac{3ma}{\lambda}}$ M1 A1 **(14)**

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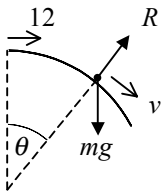
7. *(a)* just before B, resolve ↓: $60g - R_1 = 0\ \therefore R_1 = 60g$ M1

just after B, resolve ↓: $60g - R_2 = \frac{mv^2}{r} = \frac{60\times12^2}{30} = 288$ M1 A1

$\therefore R_2 = 60g - 288$ so loss of reaction = 288 N A1

(b)



resolve ↙: $mg\cos\theta - R = \frac{mv^2}{r} = \frac{mv^2}{30}$ M1 A1

at P, $R = 0\ \therefore v^2 = 30g\cos\theta$ M1

con. of ME: $\frac{1}{2}m(v^2 - 12^2) = mg \times 30(1 - \cos\theta)$ M1 A1

$\therefore v^2 = 144 + 60g(1 - \cos\theta)$ A1

combining, $v^2 = 144 + 60g(1 - \frac{v^2}{30g})$ M1

giving $v^2 = 144 + 60g - 2v^2\ \therefore 3v^2 = 144 + 60g$

so, $v^2 = 48 + 20g\ \therefore v = 15.6$ ms^{-1} (3sf) A1

(c) con. of ME: $\frac{1}{2}m(v^2 - 12^2) = mg \times 30$ M1 A1

giving $v^2 = 144 + 60g\ \therefore v = 27.1$ ms^{-1} (3sf) A1 **(15)**

Total **(75)**

Performance Record – M3 Paper C

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	elastic string, EPE	variable accel.	circular motion	centre of mass, equilm.	variable force	elastic spring, SHM	motion in a vertical circle	
Marks	7	7	9	11	12	14	15	75
Student								