GCE Examinations

Mechanics
Module M3

Advanced Subsidiary / Advanced Level

Paper B

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

When a numerical value of g is required, use $g = 9.8 \text{ m s}^{-2}$.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Shaun Armstrong & Chris Huffer

1. A student is attempting to model the expansion of an airbag in a car following a collision.

The student considers the displacement from the steering column, s metres, of a point P on the airbag t seconds after a collision and uses the formula

$$s = e^{3t} - 1, \quad 0 \leq t \leq 0.1$$

Using this model,

(a) find, correct to the nearest centimetre, the maximum displacement of P, **(2 marks)**

(b) find the initial velocity of P, **(3 marks)**

(c) find the acceleration of P in terms of t. **(1 mark)**

(d) Explain why this model is unlikely to be realistic. **(1 mark)**

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2. A particle P is attached to one end of a light elastic string of modulus of elasticity 80 N. The other end of the string is attached to a fixed point A.

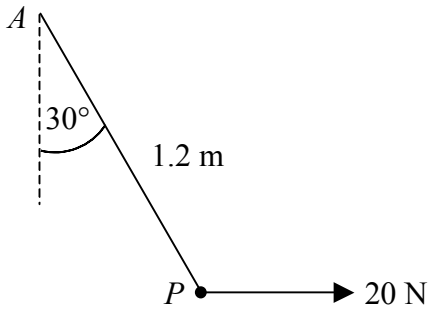

Fig. 1

When a horizontal force of magnitude 20 N is applied to P, it rests in equilibrium with the string making an angle of 30° with the vertical and $AP = 1.2$ m as shown in Figure 1.

(a) Find the tension in the string. **(3 marks)**

(b) Find the elastic potential energy stored in the string. **(5 marks)**

3.

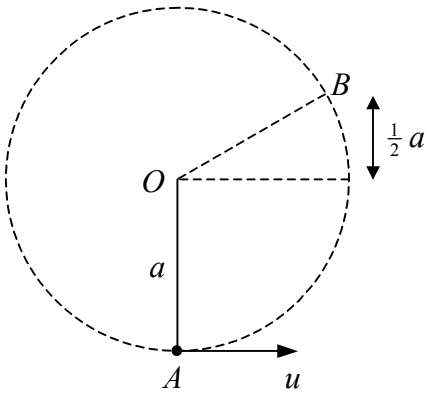

Fig. 2

A particle of mass m is suspended at a point A vertically below a fixed point O by a light inextensible string of length a as shown in Figure 2. The particle is given a horizontal velocity u and subsequently moves along a circular arc until it reaches the point B where the string becomes slack.

Given that the point B is at a height $\frac{1}{2}a$ above the level of O,

(a) show that $\angle BOA = 120°$, **(2 marks)**

(b) show that $u^2 = \frac{7}{2}ga$. **(6 marks)**

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4. On a particular day, high tide at the entrance to a harbour occurs at 11 a.m. and the water depth is 14 m. Low tide occurs $6\frac{1}{4}$ hours later at which time the water depth is 6 m.

In a model of the situation, the water level is assumed to perform simple harmonic motion.

Using this model,

(a) write down the amplitude and period of the motion. **(2 marks)**

A ship needs a depth of 9 m before it can enter or leave the harbour.

(b) Show that on this day a ship must enter the harbour by 2.38 p.m., correct to the nearest minute, or wait for low tide to pass.

(6 marks)

Given that a ship is not ready to enter the harbour until 5 p.m.,

(c) find, to the nearest minute, how long the ship must wait before it can enter the harbour.

(4 marks)

Turn over

5. *(a)* Use integration to show that the centre of mass of a uniform solid right circular cone of height h is $\frac{3}{4}h$ from the vertex of the cone.

(6 marks)

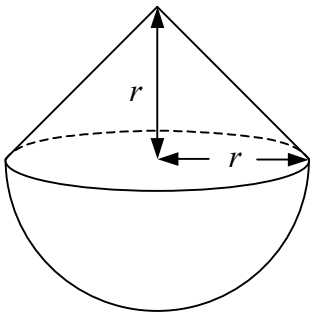

Fig. 3

A paperweight is made by removing material from the top half of a solid sphere of radius r so that the remaining solid consists of a hemisphere of radius r and a cone of height r and base radius r as shown in Figure 3.

(b) Find the distance of the centre of mass of the paperweight from its vertex. **(7 marks)**

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6. A car is travelling on a horizontal racetrack round a circular bend of radius 40 m.
The coefficient of friction between the car and the road is $\frac{2}{5}$.

(a) Find the maximum speed at which the car can travel round the bend without slipping, giving your answer correct to 3 significant figures.

(5 marks)

The owner of the track decides to bank the corner at an angle of 25° in order to enable the cars to travel more quickly.

(b) Show that this increases the maximum speed at which the car can travel round the bend without slipping by 63%, correct to the nearest whole number.

(8 marks)

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7. A particle is travelling along the x-axis. At time $t = 0$, the particle is at O and it travels such that its velocity, v m s^{-1}, at a distance x metres from O is given by

$$v = \frac{2}{x+1}.$$

The acceleration of the particle is a m s^{-2}.

(a) Show that $a = \dfrac{-4}{(x+1)^3}$. **(4 marks)**

The points A and B lie on the x-axis. Given that the particle travels d metres from O to A in T seconds and 4 metres from A to B in 9 seconds,

(b) show that $d = 1.5$, **(8 marks)**

(c) find T. **(2 marks)**

END