Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. One end of a light inextensible string of length $2r$ m is attached to a fixed point O. A particle of mass m kg is attached to the other end Q of the string, so that it can move in a vertical plane. The string is held taut and horizontal and the particle is projected vertically downwards with a speed $\sqrt{(gr)}$ ms^{-1}. When the string is vertical it begins to wrap round a small, smooth peg X at a distance r m vertically below O. The particle continues to move.

   (a) Find the speed of the particle when it reaches O, in terms of g and r. **(2 marks)**

   (b) Show that, when QX is horizontal, the tension in the string is $3mg$ N. **(5 marks)**

2. A particle moving along the x-axis describes simple harmonic motion about the origin O. The period of its motion is $\frac{\pi}{2}$ seconds. When it is at a distance 1 m from O, its speed is 3 ms^{-1}. Calculate

   (a) the amplitude of its motion, **(4 marks)**

   (b) the maximum acceleration of the particle, **(1 mark)**

   (c) the least time that it takes to move from O to a point 0·25 m from O. **(4 marks)**

3. A particle P of mass m kg is attached to the mid-point of a light elastic string of natural length $8l$ m and modulus of elasticity λ N. The two ends of the string are attached to fixed points A and B on the same horizontal level, where $AB = 8l$ m. P is released from rest at the mid-point of AB.

   (a) If P comes to instantaneous rest at a depth $3l$ m below AB, find an expression for λ in terms of m and g. **(4 marks)**

   (b) Using this value of λ, show that the speed v ms^{-1} of P when it passes through the point $2l$ m below AB is given by $v^2 = 4(24\sqrt{5} - 53)gl$. **(5 marks)**

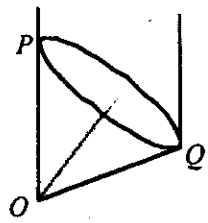
4. A particle P of mass 0·8 kg moves along a straight line OL and is acted on by a resistive force of magnitude R N directed *towards* the fixed point O. When the displacement of P from O is x m, $R = \frac{0{\cdot}8xv^2}{1+x^2}$, where v ms^{-1} is the speed of P at that instant.

   (a) Write down a differential equation for the motion of P. **(2 marks)**

   Given that $v = 2$ when $x = 0$,

   (b) find the speed with which P passes through the point A, where $OA = 1$ m. **(7 marks)**

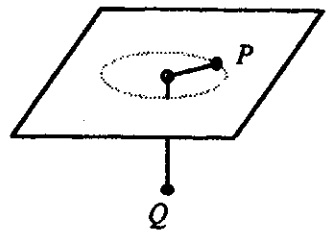
5. The diagram shows a uniform solid right circular cone of mass m kg, height h m and base radius r m suspended by two vertical strings attached to the points P and Q on the circumference of the base. The vertex O of the cone is vertically below P.



(a) Show that the tension in the string attached at Q is $\frac{3mg}{8}$ N. **(8 marks)**

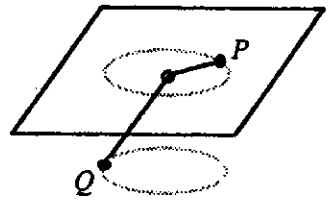
(b) Find, in terms of m and g, the tension in the other string. **(2 marks)**

6. Two identical particles P and Q are connected by a light inextensible string passing through a small smooth-edged hole in a smooth table, as shown.
P moves on the table in a horizontal circle of radius 0·2 m and Q hangs at rest.



(a) Calculate the number of revolutions made per minute by P. **(5 marks)**

Q is now also made to move in a horizontal circle of radius 0·2 m below the table. The part of the string between Q and the table makes an angle of 45° with the vertical.



(b) Show that the numbers of revolutions per minute made by P and Q respectively are in the ratio $2^{1/4} : 1$. **(9 marks)**

7. A particle P of mass m kg is fixed to one end of a light elastic string of natural length l m and modulus of elasticity kmg N. The other end of the string is fixed to a point X on a horizontal plane. P rests at O, where $OX = l$ m, with the string just taut. It is then pulled away from X through a distance $\frac{3l}{4}$ m and released from rest. On this side of O, the plane is smooth.

(a) Show that, as long as the string is taut, P performs simple harmonic motion. **(4 marks)**

(b) Given that P first returns to O with speed $\sqrt{(gl)}$ ms^{-1}, find the value of k. **(3 marks)**

(c) On the other side of O the plane is rough, the coefficient of friction between P and the plane being μ. If P does not reach X in the subsequent motion, show that $\mu > \frac{1}{2}$. **(4 marks)**

(d) If, further, $\mu = \frac{3}{4}$, show that the time which elapses after P is released and before it comes to rest is $\frac{1}{24}(9\pi + 32)\sqrt{\frac{l}{g}}$ s. **(6 marks)**

