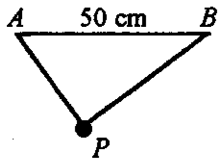
MECHANICS (A) UNIT 3 — TEST PAPER 10

Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

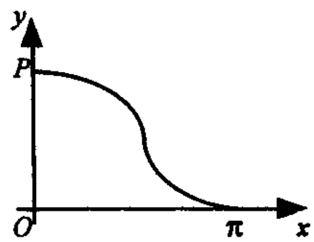
1. A cyclist travels on a banked track inclined at 8° to the horizontal. He moves in a horizontal circle of radius 10 m at a constant speed of v ms^{-1}. If there is no sideways frictional force on the cycle, calculate the value of v. **(6 marks)**

2. The figure shows a particle P, of mass 0·8 kg, attached to the ends of two light elastic strings. AP has natural length 20 cm and modulus of elasticity λ N. BP has natural length 20 cm and modulus of of elasticity μ N. A and B are fixed to points on the same horizontal level so that $AB = 50$ cm. When P is suspended in equilibrium, $AP =$ 30 cm and $BP = 40$ cm. Calculate the values of λ and μ. **(9 marks)**



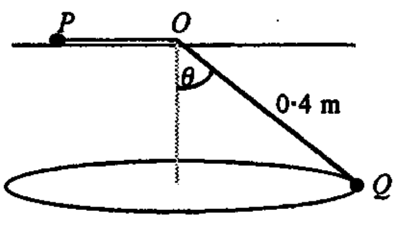
3. Suraiya, whose mass is m kg, takes a running jump into a swimming pool so that she begins to swim in a straight line with speed 0·2 ms^{-1}. She continues to move in the same straight line, the only force acting on her being a resistance of magnitude $mv^2 \sin\left(\frac{t}{100}\right)$ N, where v ms^{-1} is her speed at time t seconds after entering the pool and $0 \le t \le 50\pi$.
   (a) Find an expression for v in terms of t. **(7 marks)**
   (b) Calculate her greatest and least speeds during her motion. **(3 marks)**

4. A uniform lamina is in the shape of the region enclosed by the coordinate axes and the curve with equation $y = 1 + \cos x$, as shown.
   (a) Show by integration that the centre of mass of the lamina is at a distance $\dfrac{\pi^2 - 4}{2\pi}$ from the y-axis. **(9 marks)**



   Given that the centre of mass is at a distance 0·75 units from the x-axis, and that P is the point (0, 2) and O is the origin (0, 0),
   (b) find, to the nearest degree, the angle between the line OP and the vertical when the lamina is freely suspended from P. **(3 marks)**

5. A particle P, of mass 0·5 kg, rests on the surface of a rough horizontal table. The coefficient of friction between P and the table is 0·5. P is connected to a particle Q, of mass 0·2 kg, by a light inextensible string passing through



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a small smooth hole at a point O on the table, such that the distance OQ is 0·4 m. Q moves in a horizontal circle while P remains in limiting equilibrium.

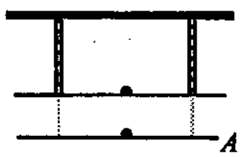
(a) Calculate the angle θ which OQ makes with the vertical. **(4 marks)**

(b) Show that the speed of Q is 1·33 ms^{-1}. **(3 marks)**

The motion is altered so that Q hangs at rest below O and P moves in a horizontal circle on the table with speed 0·84 ms^{-1}, at a constant distance r m from O but tending to slip *away* from O.

(c) Find the value of r. **(5 marks)**

6. The figure shows a swing consisting of two identical vertical light springs attached symmetrically to a light horizontal cross-bar and supported from a strong fixed horizontal beam. When a mass of 24 kg is placed at the mid-point of the cross-bar, both springs extend by 30 cm to the position A, as shown.



Each spring has natural length l m and modulus of elasticity λ N.

(a) Show that $\lambda = 392l$. **(2 marks)**

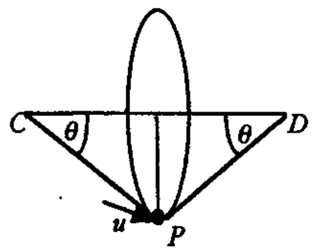
The 24 kg mass is left on the bar and the bar is then displaced downwards by a further 20 cm.

(b) Prove that the system comprising the bar and the mass now performs simple harmonic motion with the centre of oscillation at the level A. **(5 marks)**

(c) Calculate the number of oscillations made per second in this motion. **(3 marks)**

(d) Find the maximum acceleration which the mass experiences during the motion. **(2 marks)**

7. A particle P of mass m kg is attached to points C and D on the same horizontal level by means of two light inextensible strings CP and DP, both of length 40 cm. P is projected with speed u ms^{-1} so as to move in a vertical circle in a plane perpendicular to CD, so that angle PCD = angle PDC = θ throughout the motion.



If u is just large enough for the strings to remain taut as P describes this circular path,

(a) show that $u^2 = 2g \sin \theta$. **(8 marks)**

The string DP breaks when P is at its lowest point. P then immediately starts to move in a horizontal circle on the end of the string CP.

(b) Prove that $\tan \theta = \frac{1}{5}\sqrt{5}$. **(6 marks)**

