

MECHANICS 3 (A) TEST PAPER 6 : ANSWERS AND MARK SCHEME

1. (a) $K.E. = \frac{1}{2}mv^2 = \frac{1}{2}ml^2\omega^2$ J M1 A1
 (b) $P.E. = mgl(1 - \cos \theta)$ J M1 A1
 (c) Total energy $E = \frac{1}{2}ml^2\omega^2 + mgl(1 - \cos \theta)$ J M1 A1
 $\frac{dE}{dt} = mgl \sin \theta \frac{d\theta}{dt} = mgl\omega \sin \theta$ Maximum when $\theta = \frac{\pi}{2}$ A1 7
2. (a) $x = p \sin \omega t + q \cos \omega t$ $\dot{x} = p\omega \cos \omega t - q\omega \sin \omega t$ M1 A1
 $\ddot{x} = -p\omega^2 \sin \omega t - q\omega^2 \cos \omega t = -\omega^2(p \sin \omega t + q \cos \omega t) = -\omega^2 x$ M1 A1
 Acc. prop. to displacement and directed towards O , so SHM B1
 (b) Maximum speed at O , so $15 = a\omega = 3a$ $a = 5$ m M1 A1 7
3. (a) $T = 0.2(1.15)\pi^2$, $T = \frac{4}{1}(0.15)$, so $\lambda = \frac{(0.2)(1.15)\pi^2}{0.15} = 15.1$ N M1 A1 M1 A1 M1 A1
 (b) $T_1 = mg = 0.2(9.8)$ $1.96 = \frac{\lambda}{1}x$ $x = 0.1298$ m ≈ 13 cm B1 M1 A1 9
4. (a) $u =$ speed before impact. Energy : $3mgl = \frac{3mg}{4l}l^2 + \frac{1}{2}mu^2$ (1) M1 A1 A1
 $eu =$ speed after impact. Energy : $2mgl = \frac{3mg}{4l}l^2 + \frac{1}{2}me^2u^2$ (2) B1 M1 A1
 (1) gives $u^2 = \frac{9gl}{2}$, so $\frac{5}{4} = \frac{9e^2}{4}$ $9e^2 = 5$ $e = \frac{1}{3}\sqrt{5}$ M1 A1 A1
 (b) Assumption : P is a particle ; no air resistance B1 10
5. (a) When distance from surface = x m, $mv \frac{dv}{dx} = mg - v^2$ B1 M1 A1 A1
 (b) $\int \frac{mv}{mg - v^2} dv = x + c$ $\frac{1}{2} m \ln (mg - v^2) = x + c$ M1 A1 M1 A1
 $v = 0, x = 0 : c = -\frac{1}{2} m \ln mg$ $x = \frac{1}{2} m \ln \frac{mv}{mg - v^2}$ M1 A1 A1
 When $v = \sqrt{\frac{mg}{2}}$, $x = \frac{1}{2} m \ln 2$ M1 A1 13
6. (a) P moves up $r \sin \theta$ while Q moves down by arc length $r\theta$ M1 A1 A1
 Ratio of vertical distances moved = $\frac{r\theta}{r \sin \theta} = \frac{\theta}{\sin \theta}$ A1
 (b) $mg \sin \theta - R = \frac{mv^2}{r}$ Energy : $mgr\theta - mgr \sin \theta = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$ M1 A1 A1 M1 A1 A1
 $R = mg(2 \sin \theta - \theta) = mg(1 - \frac{\pi}{6})$ when $\theta = \frac{\pi}{6}$ M1 A1 A1 13
7. (a) $M(CD) : \pi r^2 h \cdot \frac{h}{2} - \frac{1}{3}\pi r^2 h \cdot \frac{3h}{4} - \frac{2}{3}\pi r^3 \cdot \frac{3r}{8} = \left(\frac{2}{3}\pi r^2 h + \frac{2}{3}\pi r^3\right) \bar{y}$ M1 A1 A1 A1
 $\frac{h^2}{2} - \frac{h^2}{4} - \frac{r^2}{4} = \frac{2}{3}(h+r)\bar{y}$ $\frac{1}{4}(h^2 - r^2) = \frac{2}{3}(h+r)\bar{y}$ M1 A1 A1
 $y = \frac{3}{8}(h-r)$ Modulus needed, since h may be $< r$ or $> r$ A1 B1
 (b) (i) $\tan \theta = \frac{3(h-r)}{8r}$ Hemisphere alone : $\tan \theta = \frac{3}{8}$ M1 A1 A1
 Thus $h - r = r$, so $h = 2r$ $h : r = 2 : 1$ A1
 (ii) C. of mass must be at O , so $y = 0$, i.e. $h = r$ $h : r = 1 : 1$ M1 A1 A1 16