

**MECHANICS 3 (A) TEST PAPER 9 : ANSWERS AND MARK SCHEME**

1. (a)  $\frac{1}{2} mu^2 = \frac{1}{2} mv^2 + mgh$   $v = 0, h = 2r : \frac{1}{2} u^2 = 2gr$   $u = 2\sqrt{gr}$  M1 A1 A1  
 (b) At top, force towards centre =  $\frac{mv^2}{r}$ , =  $mg$  as  $R = 0$  M1 A1  
 Thus  $v^2 = gr$ , so  $mu^2 = mv^2 + 4mgr = 5mgr$   $u = \sqrt{5gr}$  M1 A1 7
2. Height of each section =  $\frac{3h}{24} = \frac{h}{8}$ , mass of each section =  $\frac{3M}{24} = \frac{M}{8}$  B1 B1  
 $M(BD) : \frac{M}{4} \left(\frac{h}{8}\right) + \frac{M}{8} \left(\frac{7h}{16}\right) + \frac{M}{8} \left(\frac{11h}{16}\right) + \frac{M}{8} \left(\frac{15h}{16}\right) = \frac{5M}{8} \bar{y}$  M1 A1 A1  
 $\frac{5}{8} \bar{y} = \frac{37}{128} h$   $\bar{y} = \frac{37}{80} h (\approx 0.46h)$  M1 A1 7
3. (a) Loss in P.E. = gain in E.P.E. :  $Mge = \frac{\lambda}{2l} e$   $e = \frac{2Mgl}{\lambda}$  M1 A1 A1  
 (b) Loss in P.E. of  $A$  = gain in K.E. of  $(A \& B)$  + gain in E.P.E. M1 M1  
 $3 \times 9.8 \times 0.5 = \frac{1}{2} (4.5)v^2 + 17.5(0.25)$   $v^2 = 4.589$   $v = 2.14 \text{ ms}^{-1}$  A1 A1 M1 A1 9
4. (a)  $mv \frac{dv}{dx} = -\frac{km}{x^2}$   $\frac{v^2}{2} = \frac{k}{x} + c$   $x = a, v = 0; c = -\frac{k}{a}$  M1 A1 M1 A1  
 $\frac{v^2}{2} = k \left(\frac{1}{x} - \frac{1}{a}\right)$   $v = \sqrt{\frac{2k}{a} \left(\frac{a-x}{x}\right)}$  M1 A1  
 (b)  $v = \frac{dx}{dt}$ , so  $\frac{dx}{dt} = \sqrt{\frac{2k(a-x)}{ax}} = \sqrt{\frac{1-x}{x}}$   $\int dt = \int \sqrt{\frac{x}{1-x}} dx$  M1 A1 A1  
 $T = [\arcsin(\sqrt{x}) - \sqrt{(x-x^2)}]_{1/2}^1 = \frac{\pi}{2} - \left(\frac{\pi}{4} - \frac{1}{2}\right) = \frac{\pi+2}{4}$  M1 A1 11
5. (a) With no slip,  $R \sin 4^\circ = \frac{mu^2}{100}$ ,  $R \cos 4^\circ = mg$  B1 B1  
 $u^2 = 100g \tan 4^\circ = 68.53$   $u = 8.28$  M1 A1  
 (b)  $R \cos 4^\circ - F \sin 4^\circ = mg$   $R \sin 4^\circ + F \cos 4^\circ = \frac{m(12.5)^2}{100}$  M1 A1 M1 A1  
 Solve :  $R = 9.885m, F = 0.875m$   $F \leq \mu R$ , so  $\mu \geq F/R = 0.089$  M1 A1 A1 M1 A1 13
6. (a) Reaction  $R$  acts on  $P$  towards centre of sphere, at  $\theta$  to vertical M1  
 where  $\cos \theta = \frac{h}{r}$   $R \cos \theta = mg$ , so  $R = \frac{mgr}{h}$  B1 M1 A1  
 (b) Resolve towards centre :  $R \sin \theta = \frac{mv^2}{r \sin \theta}$  B1  
 $v^2 = \frac{mgr^2 \sin^2 \theta}{h m} = \frac{gr^2 (r^2 - h^2)}{r^2}$   $v = \sqrt{\frac{g(r^2 - h^2)}{h}}$  M1 A1 A1  
 (c)  $mg \sin \theta + S \cos \theta = mg$   $S = mg \frac{(1 - \sin \theta)}{\cos \theta} = mg \left(1 - \frac{0.866}{0.5}\right)$  M1 A1 M1  
 $= 0.268mg$   $R = mg \frac{2h}{h} = 2mg$  Hence  $S < \frac{R}{6}$  A1 A1 13
7. (a) Eqm. :  $T = F$   $\frac{\lambda}{l} \cdot \frac{l}{4} = \mu mg$   $\mu = \frac{1}{4}$  M1 A1  
 (b) At dist.  $x$ ,  $T - \mu mg = -mx$   $\frac{mg}{l} \left(\frac{l}{4} + x\right) - \frac{1}{4} mg = -mx$  M1 A1 A1  
 $x = -\frac{g}{l} x$  S.H.M. with  $\omega^2 = \frac{g}{l}$  A1  
 (c) Amplitude =  $\frac{3l}{4}$   $x = \frac{3l}{4} \cos \omega t$   $x = -\frac{l}{4} : t = \frac{1}{\omega} \arccos\left(-\frac{1}{3}\right)$  M1 A1 M1  
 $= \sqrt{\frac{l}{g}} \left(\frac{\pi}{2} + \arcsin\left(\frac{1}{3}\right)\right)$  s M1 A1  
 (d) At nat. length,  $v^2 = \frac{g}{l} \left(\frac{9l^2}{16} - \frac{l^2}{16}\right) = \frac{g}{2}$  At  $O$ ,  $v^2 = \frac{g}{2} - 2\frac{g}{4}$   $v = 0$  M1 A1 M1 A1 15