MECHANICS 3 (A) TEST PAPER 7 : ANSWERS AND MARK SCHEME

1.	$0{\cdot}27 = mr\omega^2 = 0{\cdot}6r(1{\cdot}5^2)$ $\quad r = 0{\cdot}2$ m	M1 A1 A1	3
2.	Vert. : $R + T\sin 60° = 0{\cdot}7g$ $\quad$ Horiz. : $T\cos 60° = F$	M1 A1 M1 A1	
	$F = 0{\cdot}25R$, so $0{\cdot}5T = 0{\cdot}25(6{\cdot}86 - 0{\cdot}866T)$ $\quad T = 2{\cdot}394$	M1 A1	
	$T = 6{\cdot}86x \div 0{\cdot}5$, so $x = 1{\cdot}196 \div 6{\cdot}86 = 0{\cdot}174$ m ≈ 17 cm	M1 A1	8
3.	(a) Energy conserved, so $mgl(1 + \cos 30°) = \frac{1}{2}mv^2$	M1 A1	
	Hence $v^2 = 2g(1 + \cos 30°) = g(2 + \sqrt{3})$	A1	
	$a_y = \frac{v^2}{r} = g(2 + \sqrt{3})$ towards O; $\quad a_x = 0$ (no horizontal force)	A1 A1	
	(b) At bottom, $R = \frac{mv^2}{r} + mg = mg(2 + \sqrt{3}) + mg = 0{\cdot}1g(3 + \sqrt{3})$	M1 A1 A1	8
4.	(a) $v^2 = n^2(a^2 - x^2)$ $\quad 36 = n^2(a^2 - 16)$, $16 = n^2(a^2 - 36)$	M1 A1 A1	
	$36(a^2 - 36) = 16(a^2 - 16)$ $\quad 20a^2 = 1040$ $\quad a = 7{\cdot}21$ m	M1 A1 A1	
	(b) $n^2 = 1$ $\quad n = 1$ $\quad$ Period $= \frac{2\pi}{n} = 2\pi$ s	M1 A1 A1	9
5.	(a) $x^2 + y^2 = r^2$ $\quad \bar{x}\int_0^r \pi y^2\,dx = \int_0^r \pi xy^2\,dx$	B1 M1 A1	
	$\pi\bar{x}\int_0^r r^2 - x^2\,dx = \pi\int_0^r r^2x - x^3\,dx$ $\quad \frac{2r^3}{3}\bar{x} = \frac{r^4}{4}$ $\quad \bar{x} = \frac{3r}{8}$	M1 A1 A1 A1	
	(b) M(O) : $\frac{2}{3}\pi r^3.\frac{3r}{8} = \pi\left(\frac{3r}{4}\right)^2.kr.\frac{kr}{2}$ $\quad k^2 = \frac{8}{9}$ $\quad k = \frac{2}{3}\sqrt{2}$	M1 A1 A1 A1	
	(c) $\tan\theta = \frac{3r}{4} \div \frac{2r\sqrt{2}}{3} = \frac{9}{8\sqrt{2}} = 0{\cdot}7955$ $\quad \theta = 38{\cdot}5°$	M1 A1 A1	14
6.	(a) $g = \frac{kM(1)}{R^2}$, so $k = \frac{gR^2}{M}$	M1 A1	
	(b) $\frac{mv^2}{r} = \frac{kMm}{r^2}$ $\quad v^2 = \frac{gR^2}{M}.\frac{M}{r} = \frac{gR^2}{r}$ $\quad T = \frac{2\pi r}{V} = 2\pi\sqrt{\frac{r^3}{gR^2}}$	M1 A1 M1 A1	
	(c) Diagram $\quad$ (d) Along XE, as X in circular orbit so central force	B1; B2	
	(e) $\frac{mv^2}{r} = \frac{gR^2}{M}\left(\frac{m^2}{3r^2}\cos 30^{\circ} + \frac{mM}{r^2} + \frac{m^2}{3r^2}\cos 30^{\circ}\right)$	M1 A1 A1	
	$\frac{mv^2}{r} = \frac{gR^2m}{Mr^2}\left(\frac{m}{3}\frac{\sqrt{3}}{2} \times 2 + M\right)$ $\quad v^2 = \frac{gR^2}{Mr}\left(M + \frac{m\sqrt{3}}{3}\right)$	M1 A1	
	$T_1 = \frac{2\pi r}{V} = 2\pi r\sqrt{\frac{3Mr}{gR^2(3M + m\sqrt{3})}} = 2\pi\sqrt{\frac{r^3}{gR^2}}\sqrt{\frac{3M}{3M + m\sqrt{3}}} = T\sqrt{\frac{3M}{3M + m\sqrt{3}}}$	M1 A1	16
7.	(a) In eq. position, $\frac{\lambda}{3l}l = mg$ $\quad \lambda = 3mg$	M1 A1	
	At depth x below eq. position, $mg - T = m\ddot{x}$	M1	
	$mg - \frac{3mg}{3l}(l + x) = m\ddot{x}$ $\quad \ddot{x} = -\frac{g}{l}x$ $\quad$ SHM, with $\omega^2 = \frac{g}{l}$	A1 A1	
	(b) $x = 2l\cos\omega t$ $\quad$ When $x = -l$, $\omega t = \frac{2\pi}{3}$ $\quad t = \frac{2\pi}{3\omega} = \frac{2\pi}{3}\sqrt{\frac{l}{g}}$	B1 M1 A1 A1	
	(c) When released, E.P.E. $= \frac{3mg(9l^2)}{2(3l)} = \frac{9mgl}{2}$	M1 A1	
	At max. height H, P.E. $= mgH = \frac{9mgl}{2}$ $\quad H = \frac{9l}{2}$	M1 A1	
	(d) When slack, $v^2 = 3gl$ $\quad 0 = \sqrt{(3gl)} - gt_H$ $\quad t_H = \sqrt{\frac{3l}{g}}$ $\quad T_h = \frac{2\pi}{3}\sqrt{\frac{l}{g}} + \sqrt{3}\sqrt{\frac{l}{g}}$	M1 A1 M1 A1	17