MECHANICS 3 (A) TEST PAPER 1 : ANSWERS AND MARK SCHEME

1. $L\sin\theta = mv^2/r = 0{\cdot}5 \times 36 \div 2 = 9$ $\quad L\cos\theta = 0{\cdot}5g = 4{\cdot}9$ M1 A1 M1 A1
   $\tan\theta = 9 \div 4{\cdot}9 = 1{\cdot}84$ $\quad \theta = 61{\cdot}4^\circ \approx 61^\circ$ M1 A1 A1 — 7

2. (a) $T = \frac{\lambda(0{\cdot}1)}{0{\cdot}2} = \frac{\lambda}{2}$ $\quad$ Resolve vertically : $2T\cos 30^\circ = 0{\cdot}7g$ M1 A1 M1 A1
   $T = 3{\cdot}96$ $\quad \lambda = 2T = 7{\cdot}92$ M1 A1
   (b) Assumed P is a particle, e.g. a single point at vertex of the Δ B1 — 7

3. $F = ma$: $0{\cdot}5v\frac{dv}{dx} = -\frac{8}{x^2}$ $\quad \int v\,dv = -\int \frac{16}{x^2}\,dx$ B1 M1
   $\frac{v^2}{2} = \frac{16}{x} + c$ $\quad x = 2, v = 4$: $8 = 8 + c$ $\quad c = 0$ $\quad v^2 = \frac{32}{x}$ A1 M1 A1 A1
   When $x = 0{\cdot}5$, $v^2 = 64$ $\quad |v| = 8\ \text{ms}^{-1}$ M1 A1 — 8

4. (a) At greatest depth, gravitational P.E. lost = elastic P.E. gained
   $mg(2l) = \lambda l^2/2l$ $\quad \lambda = 4mg$ M1 M1 A1
   (b) Gravitational P.E. lost = elastic P.E. gained + K.E. gained
   $mg\frac{5l}{4} = \frac{4mg}{2l}\cdot\frac{l^2}{16} + \frac{1}{2}mv^2$ $\quad \frac{5gl}{4} - \frac{gl}{8} = \frac{1}{2}v^2$ M1 A1 A1
   $v^2 = \frac{9gl}{4}$ $\quad v = \frac{3}{2}\sqrt{gl}\ \text{ms}^{-1}$ M1 A1 A1 — 9

5. (a) Let mass of cone = M, so mass removed = $\frac{1}{8}M$, remainder = $\frac{7}{8}M$ M1 A1
   $M\frac{h}{4} = \frac{M}{8}\left(\frac{h}{2} + \frac{h}{8}\right) + \frac{7M}{8}\bar{y}$ $\quad \frac{7}{8}\bar{y} = \frac{h}{4} - \frac{5h}{64} = \frac{11h}{64}$ $\quad \bar{y} = \frac{11h}{56}$ M1 A1 A1 M1 A1
   (b) $\tan\alpha = \frac{r}{2} \div \left(\frac{h}{2} - \frac{11h}{56}\right) = \frac{28r}{17h}$ $\quad \frac{28r}{17h} = \frac{1}{2}$ $\quad h : r = 56 : 17$ M1 A1 M1 A1 A1 — 12

6. (a) $mg = \frac{mg}{2l}e$ $\quad e : l = 2 : 1$ M1 A1
   (b) $m\ddot{x} = mg - \frac{mg}{2l}(2l + x)$ $\quad \ddot{x} = -\frac{g}{2l}x$, so S.H.M. M1 A1 M1 A1
   (c) $\omega^2 = \frac{g}{2l}$ $\quad$ Period $= \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2l}{g}}$ s M1 A1
   (d) At E, $v = a\omega = \frac{3l}{2}\sqrt{\frac{g}{2l}} = \sqrt{\frac{9gl}{8}}\ \text{ms}^{-1}$ M1 A1
   (e) $x = \frac{3l}{2}\cos\omega t$ $\quad x = -\frac{3l}{4}$ when $-\frac{3l}{4} = \frac{3l}{2}\cos\omega t$ M1 A1 A1
   $\cos\omega t = -\frac{1}{2}$ $\quad \omega t = \frac{2\pi}{3}$ $\quad t = \frac{2\pi}{3\omega} = \frac{2\pi}{3}\sqrt{\frac{2l}{g}}$ s M1 A1 — 15

7. (a) K.E. lost = P.E. gained : $\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mgl - mgl\cos\theta$ M1 A1
   $v^2 = u^2 - 2gl + 2gl\cos\theta$ A1
   $T - mg\cos\theta = \frac{mv^2}{l}$ $\quad T = \frac{m}{l}(u^2 - 2gl + 2gl\cos\theta) + mg\cos\theta$ M1 A1 M1 A1
   $T = 0$ when $\theta = 120^\circ$: $\frac{u^2}{l} - 2g - g - \frac{g}{2} = 0$ $\quad u^2 = \frac{7gl}{2}$ M1 A1 A1
   (b) Now moves as projectile with initial speed v, where $v^2 = u^2 - 3gl$ M1
   so $v = \sqrt{\frac{gl}{2}}$, and angle of projection 60° A1 B1
   At highest point above O, $h = \frac{l}{2} + \frac{v^2\sin 60}{2g} = \frac{l}{2} + \frac{gl}{4g}\cdot\frac{3}{4} = \frac{11l}{16}$ m M1 A1 M1 A1 — 17