

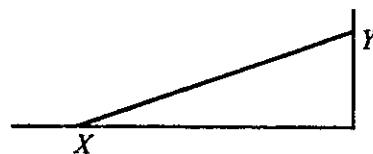
Take $g = 9.8 \text{ ms}^{-2}$ and give all answers correct to 3 significant figures where necessary.

1. A small ball A is moving with velocity $(7\mathbf{i} + 12\mathbf{j}) \text{ ms}^{-1}$. It collides in mid-air with another ball B , of mass 0.4 kg , moving with velocity $(-\mathbf{i} + 7\mathbf{j}) \text{ ms}^{-1}$. Immediately after the collision, A has velocity $(-3\mathbf{i} + 4\mathbf{j}) \text{ ms}^{-1}$ and B has velocity $(6.5\mathbf{i} + 13\mathbf{j}) \text{ ms}^{-1}$.

Calculate the mass of A .

(4 marks)

2. A stick of mass 0.75 kg is at rest with one end X on a rough horizontal floor and the other end Y leaning against a smooth vertical wall. The coefficient of friction between the stick and the floor is 0.6 . Modelling the stick as a uniform rod, find the smallest angle that the stick can make with the floor before it starts to slip.



(6 marks)

3. An engine of mass $20\,000 \text{ kg}$ climbs a hill inclined at 10° to the horizontal. The total non-gravitational resistance to its motion has magnitude $35\,000 \text{ N}$ and the maximum speed of the engine on the hill is 15 ms^{-1} .

(a) Find, in kW, the maximum rate at which the engine can work.

(4 marks)

(b) Find the maximum speed of the engine when it is travelling on a horizontal track against the same non-gravitational resistance as before.

(3 marks)

4. Relative to a fixed origin O , the points X and Y have position vectors $(4\mathbf{i} - 5\mathbf{j}) \text{ m}$ and $(12\mathbf{i} + \mathbf{j}) \text{ m}$ respectively, where \mathbf{i} and \mathbf{j} are perpendicular unit vectors in the directions due east and due north respectively. A particle P starts from X , and t seconds later its position vector relative to O is $(2t + 4)\mathbf{i} + (kt^2 - 5)\mathbf{j}$.

(a) Find the value of k if P takes 4 seconds to reach Y .

(3 marks)

(b) Show that P has constant acceleration and find the magnitude and direction of this acceleration.

(4 marks)

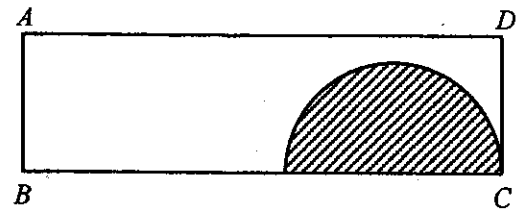
5. Three particles A , B and C , of equal size and each of mass m , are at rest on the same straight line on a smooth horizontal surface. The coefficient of restitution between A and B , and between B and C , is e .

A is projected with speed 7 ms^{-1} and strikes B directly. B then collides with C , which starts to move with speed 4 ms^{-1} .

Calculate the value of e .

(10 marks)

6. A rectangular piece of cardboard $ABCD$, measuring 30 cm by 12 cm, has a semicircle of radius 5 cm removed from it as shown.



- (a) Calculate the distances of the centre of mass of the remaining piece of cardboard from AB and from BC . (7 marks)

The remaining cardboard is suspended from A and hangs in equilibrium.

- (b) Find the angle made by AB with the vertical. (4 marks)

7. A rocket is fired from a fixed point O . During the first phase of its motion its velocity, v ms^{-1} , is given at time t seconds after firing by the formula

$$v = pt^2 + qt.$$

5 seconds after firing, the rocket is travelling at 500 ms^{-1} .

30 seconds after firing, the rocket is travelling at $12\,000 \text{ ms}^{-1}$.

- (a) Find the constants p and q . (4 marks)
(b) Sketch a velocity-time graph for the rocket for $0 \leq t \leq 30$. (2 marks)
(c) Find the initial acceleration of the rocket. (2 marks)
(d) Find the distance of the rocket from O 30 seconds after firing. (4 marks)

From time $t = 30$ onwards, the rocket maintains a constant speed of $12\,000 \text{ ms}^{-1}$.

- (e) Find the average speed of the rocket during its first 50 seconds of motion. (3 marks)

8. A golf ball is hit with initial velocity $u \text{ ms}^{-1}$ at an angle of 45° above the horizontal. The ball passes over a building which is 15 m tall at a distance of 30 m horizontally from the point where the ball was hit.

- (a) Find the smallest possible value of u . (7 marks)

When u has this minimum value,

- (b) show that the ball does not rise higher than the top of the building. (4 marks)
(c) Deduce the total horizontal distance travelled by the ball before it hits the ground. (2 marks)
(d) Briefly describe two modelling assumptions that you have made. (2 marks)