GCE Examinations

Mechanics
Module M2

Advanced Subsidiary / Advanced Level

Paper D

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

When a numerical value of g is required, use $g = 9.8 \text{ m s}^{-2}$.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Shaun Armstrong & Chris Huffer

1. A particle P moves such that at time t seconds its position vector, **r** metres, relative to a fixed origin O is given by

$$\mathbf{r} = (\tfrac{3}{2}t^2 - 3t)\mathbf{i} + (\tfrac{1}{3}t^3 - kt)\mathbf{j},$$

where k is a constant and **i** and **j** are perpendicular horizontal unit vectors.

(a) Find an expression for the velocity of P at time t. **(3 marks)**

(b) Given that P comes to rest instantaneously, find the value of k. **(3 marks)**

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2. Two smooth spheres P and Q of equal radius and of mass $2m$ and $5m$ respectively, are moving towards each other along a horizontal straight line when they collide. After the collision, P and Q travel in opposite directions with speeds of 3 m s^{-1} and 4 m s^{-1} respectively.

Given that the coefficient of restitution between the two particles is $\frac{1}{2}$, find the speeds of P and Q before the collision.

(6 marks)

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3. A car of mass 1200 kg experiences a resistance to motion, R newtons, which is proportional to its speed, v m s^{-1}. When the power output of the car engine is 90 kW and the car is travelling along a horizontal road, its maximum speed is 50 m s^{-1}.

(a) Show that $R = 36v$. **(4 marks)**

The car ascends a hill inclined at an angle θ to the horizontal where $\sin\theta = \frac{1}{14}$.

(b) Find, correct to 3 significant figures, the maximum speed of the car up the hill assuming that the power output of the engine is unchanged.

(6 marks)

4.

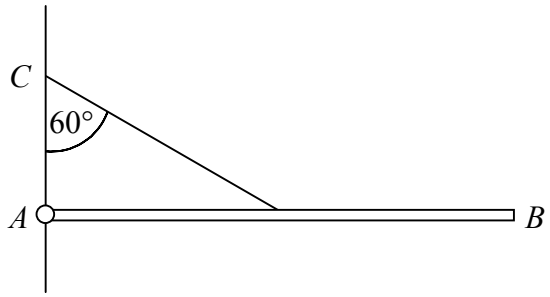

Fig. 1

Figure 1 shows a uniform rod AB of mass 2 kg and length $2a$. The end A is attached by a smooth hinge to a fixed point on a vertical wall so that the rod can rotate freely in a vertical plane. A mass of 6 kg is placed at B and the rod is held in a horizontal position by a light string joining the midpoint of the rod to a point C on the wall, vertically above A. The string is inclined at an angle of 60° to the wall.

(a) Show that the tension in the string is $28g$. **(4 marks)**

(b) Find the magnitude and direction of the force exerted by the hinge on the rod, giving your answers correct to 3 significant figures. **(8 marks)**

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5. A particle P moves in a straight line with an acceleration of $(6t - 10)$ m s^{-2} at time t seconds. Initially P is at O, a fixed point on the line, and has velocity 3 m s^{-1}.

(a) Find the values of t for which the velocity of P is zero. **(6 marks)**

(b) Show that, during the first two seconds, P travels a distance of $6\frac{26}{27}$ m. **(7 marks)**

Turn over

6.

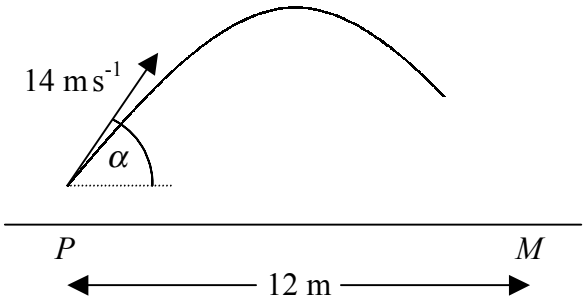

Fig. 2

A football player strikes a ball giving it an initial speed of 14 m s^{-1} at an angle α to the horizontal as shown in Figure 2. At the instant he strikes the ball it is 0.6 m vertically above the point P on the ground. The trajectory of the ball is in a vertical plane containing P and M, the middle of the goal-line. The distance between P and M is 12 m and the ground is horizontal.

Given that the ball passes over the point M without bouncing,

(a) find, to the nearest degree, the minimum value of α. **(8 marks)**

Given that the crossbar of the goal is 2.4 m above M and that $\tan\alpha = \frac{4}{3}$,

(b) show that the ball passes 4.2 m vertically above the crossbar. **(6 marks)**

7.

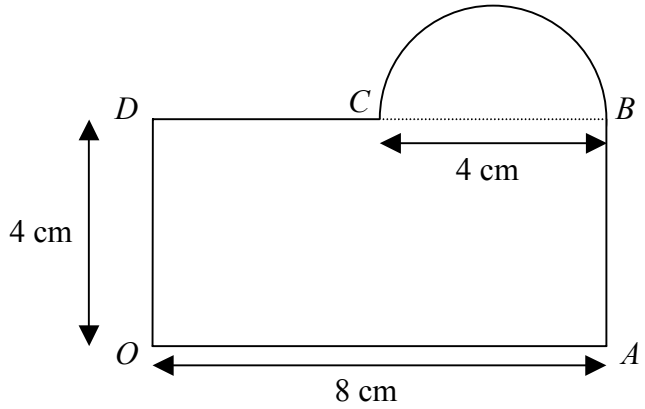

Fig. 3

Figure 3 shows a hotel 'key' consisting of a rectangle $OABD$, where $OA = 8$ cm and $OD = 4$ cm, joined to a semicircle whose diameter BC is 4 cm long. The thickness of the key is negligible and the same material is used throughout.

The key is modelled as a uniform lamina.

Using this model,

(a) find, correct to 3 significant figures, the distance of the centre of mass from

(i) OD,

(ii) OA. **(10 marks)**

A small circular hole of negligible diameter is made at the mid-point of BC so that the key can be hung on a smooth peg. When the key is freely suspended from the peg,

(b) find, correct to 3 significant figures, the acute angle made by OA with the vertical.

(4 marks)

END