Paper Reference(s)

6678/01

Edexcel GCE

Mechanics M2
Silver Level S2

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
69	62	51	41	33	25

1. A car of mass 1500 kg is moving up a straight road, which is inclined at an angle θ to the horizontal, where $\sin \theta = \frac{1}{14}$. The resistance to the motion of the car from non-gravitational forces is constant and is modelled as a single constant force of magnitude 650 N. The car's engine is working at a rate of 30 kW.

Find the acceleration of the car at the instant when its speed is 15 m s^{-1}.

(5)

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2. A particle of mass 2 kg is moving with velocity $(5\mathbf{i} + \mathbf{j})$ m s^{-1} when it receives an impulse of $(-6\mathbf{i} + 8\mathbf{j})$ N s. Find the kinetic energy of the particle immediately after receiving the impulse.

(5)

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3. A particle of mass 0.5 kg is projected vertically upwards from ground level with a speed of 20 m s^{-1}. It comes to instantaneous rest at a height of 10 m above the ground. As the particle moves it is subject to air resistance of constant magnitude R newtons. Using the work-energy principle, or otherwise, find the value of R.

(6)

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4. A particle P moves along the x-axis in a straight line so that, at time t seconds, the velocity of P is v m s^{-1}, where

$$v = \begin{cases} 10t - 2t^2, & 0 \leq t \leq 6, \\ \dfrac{-432}{t^2}, & t > 6. \end{cases}$$

At $t = 0$, P is at the origin O. Find the displacement of P from O when

(*a*) $t = 6$,

(3)

(*b*) $t = 10$.

(5)

5.

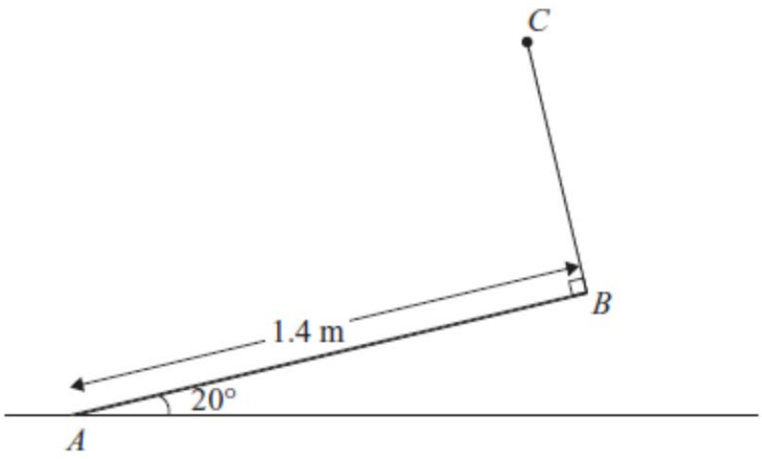

Figure 2

A uniform rod *AB* has mass 4 kg and length 1.4 m. The end *A* is resting on rough horizontal ground. A light string *BC* has one end attached to *B* and the other end attached to a fixed point *C*. The string is perpendicular to the rod and lies in the same vertical plane as the rod. The rod is in equilibrium, inclined at 20° to the ground, as shown in Figure 2.

(*a*) Find the tension in the string.

(4)

Given that the rod is about to slip,

(*b*) find the coefficient of friction between the rod and the ground.

(7)

6.

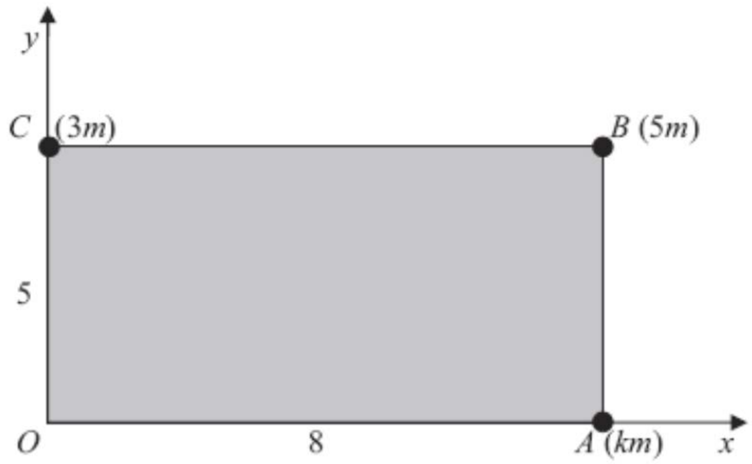

Figure 3

Figure 3 shows a rectangular lamina *OABC*. The coordinates of *O*, *A*, *B* and *C* are (0, 0), (8, 0), (8, 5) and (0, 5) respectively. Particles of mass km, $5m$ and $3m$ are attached to the lamina at *A*, *B* and *C* respectively.

The x-coordinate of the centre of mass of the three particles *without the lamina* is 6.4.

(a) Show that $k = 7$.

(4)

The lamina *OABC* is uniform and has mass $12m$.

(b) Find the coordinates of the centre of mass of the combined system consisting of the three particles and the lamina.

(6)

The combined system is freely suspended from *O* and hangs at rest.

(c) Find the angle between *OC* and the horizontal.

(3)

7. [*In this question, the unit vectors* **i** *and* **j** *are horizontal and vertical respectively*.]

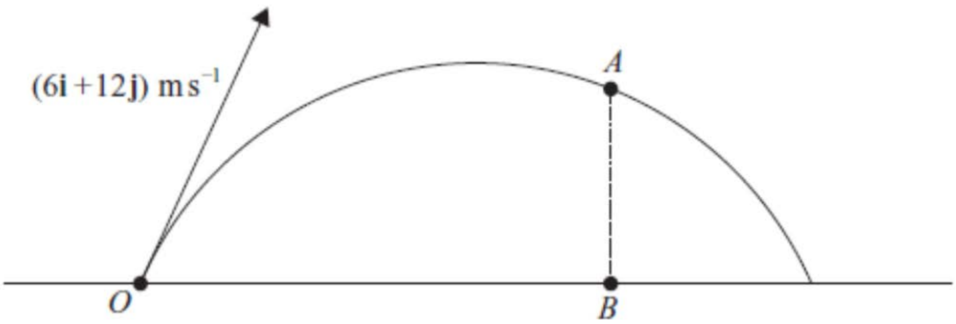

Figure 3

The point O is a fixed point on a horizontal plane. A ball is projected from O with velocity $(6\mathbf{i} + 12\mathbf{j})$ m s^{-1}, and passes through the point A at time t seconds after projection. The point B is on the horizontal plane vertically below A, as shown in Figure 3. It is given that $OB = 2AB$.

Find

(*a*) the value of t,

(7)

(*b*) the speed, V m s^{-1}, of the ball at the instant when it passes through A.

(5)

At another point C on the path the speed of the ball is also V m s^{-1}.

(*c*) Find the time taken for the ball to travel from O to C.

(3)

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8. Particles A, B and C of masses $4m$, $3m$ and m respectively, lie at rest in a straight line on a smooth horizontal plane with B between A and C. Particles A and B are projected towards each other with speeds u m s^{-1} and v m s^{-1} respectively, and collide directly. As a result of the collision, A is brought to rest and B rebounds with speed kv m s^{-1}. The coefficient of restitution between A and B is $\frac{3}{4}$.

(*a*) Show that $u = 3v$.

(6)

(*b*) Find the value of k.

(2)

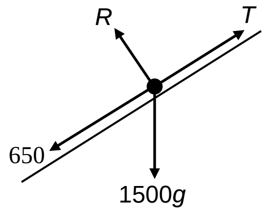
Immediately after the collision between A and B, particle C is projected with speed $2v$ m s^{-1} towards B so that B and C collide directly.

(*c*) Show that there is no further collision between A and B.

(4)

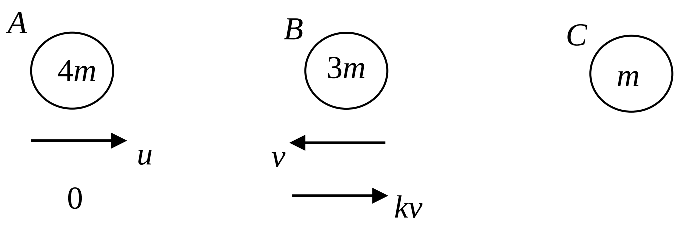
TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	 F = ma parallel to the slope, $T - 1500g\sin\theta - 650 = 1500a$ Tractive force, 30000 = T x 15 $a = \dfrac{\frac{30000}{15} - 1500(9.8)(\frac{1}{14}) - 650}{1500}$ <u>0.2</u> (m s^{-2})	**M1*** **A1** **M1*** **d*M1** **A1** **[5]**
2.	$\mathbf{I} = (-6\mathbf{i} + 8\mathbf{j}) = 2(\mathbf{v} - (5\mathbf{i} + \mathbf{j}))$ $-3\mathbf{i} + 4\mathbf{j} = \mathbf{v} - 5\mathbf{i} - \mathbf{j}$ $\mathbf{v} = 2\mathbf{i} + 5\mathbf{j}$ KE = $\frac{1}{2} \times 2 \times \left\|v^2\right\| = \left(\sqrt{2^2 + 5^2}\right)^2 = 29$(j)	**M1 A1** **A1** **M1 A1** **[5]**
3.	$\frac{1}{2}$ x 0.5 x 20^2 ; 0.5g x 10 $10R = \frac{1}{2}$ x 0.5 x $20^2 - 0.5g$ x 10 $\Rightarrow R = 5.1$	**B1 B1** **M1 A1** **DM1 A1** **[6]**

Question Number	Scheme	Marks
4 (a)	$v = 10t - 2t^2,\ s = \int v\,dt$	M1
	$= 5t^2 - \frac{2t^3}{3} (+C)$	A1
	$t = 6 \Rightarrow s = 180 - 144 = \underline{36}$ (m)	A1
		(3)
(b)	$\underline{s} = \int v\,dt = \underline{\frac{-432t^{-1}}{-1}} \left(+K\right) = \underline{\frac{432}{t}} \left(+K\right)$	<u>B1</u>
	t = 6, s = "36" $\Rightarrow 36 = \frac{432}{6} + K$	M1*
	$\Rightarrow K = -36$	A1
	At $t = 10$, $s = \frac{432}{10} - 36 = \underline{7.2}$ (m)	d*M1
		<u>A1</u>
		(5)
		[8]
5.		
(a)	Taking moments about A:	M1
	$4g \times 0.7 \times \cos 20° = 1.4T$	A1 A1
	T = 18.4 N	A1
		(4)
(b)	$\uparrow\ R + T\cos 20 = 4g$	
	$R = 4g - T\cos 20°$	M1 A1
	$\rightarrow\ F = T\sin 20$	M1 A1
	$F = \mu R \Rightarrow T\sin 20° = \mu\left(4g - T\cos 20°\right)$	M1 A1
	$\mu = \frac{T\sin 20°}{4g - T\cos 20°} = 0.29$	A1
		(7)
		(11 marks)

Question Number	Scheme	Marks
6.(a)	$M(Oy)$ $\quad (8+k)m\times 6.4 = 5m\times 8 + km\times 8$	**M1 A1**
	$1.6k = 11.2 \quad \Rightarrow \quad k = 7$ * $\qquad$ cso	**M1 A1 (4)**
(b)	$M(Oy)$ $\quad 27m\bar{x} = 12m\times 4 + 5m\times 8 + 7m\times 8$	**M1 A1**
	$\bar{x} = \frac{16}{3}$ $\qquad$ 5.3 or better	**A1**
	$M(Ox)$ $\quad 27m\bar{y} = 12m\times 2.5 + 8m\times 5$	**M1 A1**
	$\bar{y} = \frac{70}{27}$ $\qquad$ 2.6 or better	**A1 (6)**
(c)	$\tan\theta = \frac{\bar{y}}{\bar{x}} = \frac{35}{72}$	**M1 A1ft**
	$\theta \approx 26°$ $\qquad$ awrt 25.9°	**A1 (3)**
		(13 marks)
7. (a)	**i** $\rightarrow$ distance = $6t$	B1
	j $\uparrow$ distance = $12t - \frac{1}{2}gt^2$	M1 A1
	At B, $2\left(12t - \frac{1}{2}gt^2\right) = 6t$	M1 A1
	$(24-6)t = gt^2$	M1
	$18 = gt\ ,\ t = \frac{18}{g}(=1.84\text{s})$	A1
		(7)
(b)	**i** $\rightarrow$ speed = 6	B1
	j $\uparrow$ velocity = $12 - gt$ = -6	M1 A1
	$\therefore$ speed at A	
	$= \sqrt{6^2+6^2} = \sqrt{72} = 6\sqrt{2}(=8.49)(\text{ms}^{-1})$	M1 A1
		(5)
(c)	$\uparrow$ speed = $12 - gt$ = +6	M1 A1 ft
	$t = \frac{6}{g}(=0.61\text{s})$	A1
		(3)
		(15 marks)

8. (a)		
	Conservation of momentum: $4mu - 3mv = 3mkv$	M1A1
	Impact law: $kv = \frac{3}{4}(u+v)$	M1A1
	Eliminate k: $4mu - 3mv = 3m \times \frac{3}{4}(u+v)$	DM1
	$u = 3v$ (Answer given)	A1
		(6)
(b)	$kv = \frac{3}{4}(3v+v)$, $k = 3$	M1,A1
		(2)
(c)	Impact law: $(kv+2v)e = v_C - v_B$ $\quad(5ve = v_C - v_B)$	B1
	Conservation of momentum : $3 \times kv - 1 \times 2v = 3v_B + v_c$ $\quad(7v = 3v_B + v_c)$	B1
	Eliminate v_C : $v_B = \frac{v}{4}(7-5e) > 0$ hence no further collision with *A*.	M1 A1
		(4)
		[12]

Examiner reports

Question1

This question was tackled confidently and successfully by the majority of candidates. The solution was often broken down into several small steps and only put together using Newton’s second law right at the end. Sign errors were rare and resolving errors even more so. A few candidates muddled the driving force with the resultant force, or ignored the 650 N, and hence scored few marks. There were also some candidates confused about g, omitting it in the weight term and/or including it in the mass term

Question 2

This was well answered by the majority of candidates, many of whom gained full marks. Most equated impulse to change in momentum, but the subtraction was sometimes done the wrong way round. A few candidates made errors due to the poor use of brackets. Most candidates went on to find the kinetic energy correctly. Calculation of the magnitude of the velocity was often correct, but there were arithmetical errors and slips such as squaring a component twice. Some candidates did not appear to understand that kinetic energy is a scalar and gave their answer in terms of **i** and **j**. $\sqrt{29}$ and 3 were common incorrect answers.

Question 3

Most candidates showed a sound understanding of the mechanics involved in this question and gave a completely correct solution. Although candidates were allowed the choice of method, the work energy method was by far the most popular approach. Some candidates went wrong by considering both the change in potential energy and the work done against the weight. Some were clearly confused, and offered inconsistent equations involving forces and energy, usually because they had omitted the distance when considering the work done against the resistance.

The alternative method of finding the acceleration and then using $F = ma$ was equally successful. The most common error was an incorrect sign in the equation of motion.

Question 4

A few candidates were clearly confused by velocity being defined in terms of two separate functions. Nevertheless, virtually all candidates knew they had to integrate the relevant expression for velocity in order to find the displacement and they did this correctly in part (a). As the constant was zero in this part of the question, candidates who had overlooked it were not penalised. There were occasional mistakes such as differentiating instead of integrating, and some candidates who tried to use the equations for constant acceleration.

In part (b), although most correctly integrated the expression, for those that went along the indefinite integral route, the constant of integration was often just assumed to be zero because the displacement was zero at the start. Several candidates even demonstrated that the constant of integration was zero, apparently having no problem with equating 432/0 to zero! These candidates clearly did not realise that the expression was not relevant at the start. Those who found the definite integral were generally more successful. Other errors in part (b) included using $t = 4$, using $t = 7$ as a lower limit for the second integral (apparently not recognising the continuous nature of time), or reaching the correct solution but then adding the answer from (a) a second time.

Question 5

This was a similar question to those set on this topic in the recent past and was well answered by the majority of candidates. More students than usual were finding the quickest way to get the answers. The friction was acting in the right direction and nearly all normal reactions were normal.

Part (a): Most candidates started by taking moments about *A*. The most common error was to resolve the tension but then only consider either the horizontal or the vertical component. Some candidates gave a final answer with more than three significant figures, which was inappropriate following the substitution of a value for *g*.

Part (b): Most candidates chose to resolve in two directions and were able to combine their values to gain a figure for the coefficient of friction. Those who resolved vertically and horizontally had a simpler task and were nearly always successful. Resolving parallel to and/or perpendicular to the rod was less successful - frequently resulting in equations that omitted the friction at *A*. Similarly, those candidates who chose to take moments about *B* often failed to reach the correct answer because they had left out the weight or the friction at *A*.

Question 6

This question was often answered very well. Most candidates took moments about *Oy* or *Ox*, although alternatives were seen. Weaker candidates did not seem to be very confident in dealing with a lamina plus a set of particles.

(a) Full marks were usually seen. The alternative method of taking moments about axes through the centre of mass was seen and was usually implemented successfully.
A small number were too casual in claiming the printed result from a correct moment's equation – candidates need to remember that with a given answer a little more detail is required.

(b) This was usually correct. Many candidates calculated the *y* coordinate of the centre of mass of the three particles as $\frac{8}{3}$ and then used that to calculate the centre of mass of the system.
Any errors were usually in the total mass (e.g. taken to be $40m$ or equal to the total mass of only those which contributed to the moment). A few responses did not include the lamina and scored nothing. Some candidates treated *OABC* as a set of connected rods, rather than as a lamina.

(c) Candidates who answered part (b) incorrectly often picked up two marks here from the follow through. The majority of candidates recognised the correct triangle, although many of them then calculated the incorrect angle. A number of responses used 5 and / or 8, which scored nothing. A few candidates lost the final mark by using rounded values from part (b) and arrived at an incorrect value for the angle.

Question 7

Part (a): A great variety of approaches was in evidence. Most candidates were comfortable with the vector format of the question and set about considering horizontal and vertical components of the motion. The simplest approach of working with the time rather than going via calculation of the distance *AB* or *OB* was more often successful. A small minority confused the **i** and **j** components of velocity, effectively taking **i** as vertical and **j** as horizontal, and some misinterpreted the relationship $OB = 2AB$. A common error was to over specify the final answer, or to imply an exact answer of $\frac{90}{49}$.

It was disappointing to see several candidates trying to force the problem back into a non-vector form by calculating the angle of projection and then resolving. A few of these candidates were successful, despite using an unnecessarily complicated method.

Part (b): Nearly all attempted to find the vertical component of velocity using *suvat.* Some stopped at this stage, but most went on to state the horizontal component of velocity and to find the speed. Occasionally an error was generated by using their rounded answer of 1.8 from part (a) in calculations in (b). Candidates should avoid the temptation to use approximate values in their working too early. The use of $t = \frac{18}{g}$ here would have led to simpler equations and more accurate answers.

A few candidates tried to solve the problem in a multipart style, by considering motion to the maximum height and then motion from the maximum height, but were rarely successful and their multipart answers were not clearly labelled neither was their approach explained.

The energy method was occasionally employed, usually successfully.

Part (c): It is pleasing to note that many candidates were able to use the symmetry of the path to deduce that the vertical component of the velocity at *C* is +6; this then gave a quick solution to the problem. Some candidates confused their answer to part (b) with the vertical component in part (c).

Longer alternative methods, usually involving vertical distance, were also seen, but all too often accuracy errors arising from premature approximation led to a loss of marks.

Question 8

Parts (a) and (b) were tackled with confidence by most candidates although a few were not sufficiently careful with signs and/or had long-winded algebraic manipulation to achieve the given result. CLM and the impact law were generally used correctly.

Part (c) proved to be more challenging and differentiated between the stronger candidates and those with confused concepts. No mention of *e* in the question caused some to ignore it or, more commonly, to assume the previous value. There were various arguments used to justify their final statement but, encouragingly, the range for *e* seemed to be understood.

Statistics for M2 Practice Paper Silver 2

				Mean average scored by candidates achieving grade:							
Qu	**Max Score**	**Modal score**	**Mean %**	**ALL**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
1	5		85.2	4.26		4.66	3.99	3.63	2.73	2.19	0.95
2	5		84.2	4.21	4.70	4.49	4.00	3.50	3.04	2.14	1.56
3	6		77.2	4.63		5.11	4.25	3.66	2.76	2.17	1.87
4	8		68.4	5.47		6.25	4.58	4.17	3.78	2.37	1.65
5	11		73.4	8.07	10.19	9.25	6.48	5.32	3.49	2.88	1.17
6	13		77.4	10.06		11.72	10.37	9.24	7.79	5.82	3.82
7	15		70.1	10.51	13.45	11.83	8.43	6.10	4.29	2.72	0.87
8	12		69.0	8.28		9.82	8.17	7.03	5.78	4.32	2.42
	75		**74.0**	**55.49**		**63.13**	**50.27**	**42.65**	**33.66**	**24.61**	**14.31**