Paper Reference(s)

6677/01

Edexcel GCE

Mechanics M2

Bronze Level B4

Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Green)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of g is required, take g = 9.8 m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
72	63	53	43	33	26

1. A tennis ball of mass 0.1 kg is hit by a racquet. Immediately before being hit, the ball has velocity $30\mathbf{i}$ m s^{-1}. The racquet exerts an impulse of $(-2\mathbf{i} - 4\mathbf{j})$ N s on the ball. By modelling the ball as a particle, find the velocity of the ball immediately after being hit.

(4)

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2. A particle P is moving in a plane. At time t seconds, P is moving with velocity $\mathbf{v}$ m s^{-1}, where $\mathbf{v} = 2t\mathbf{i} - 3t^2\,\mathbf{j}$.

Find

(*a*) the speed of P when $t = 4$,

(2)

(*b*) the acceleration of P when $t = 4$.

(3)

Given that P is at the point with position vector $(-4\mathbf{i} + \mathbf{j})$ m when $t = 1$,

(*c*) find the position vector of P when $t = 4$.

(5)

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3. A car of mass 1000 kg is moving at a constant speed of 16 m s^{-1} up a straight road inclined at an angle θ to the horizontal. The rate of working of the engine of the car is 20 kW and the resistance to motion from non-gravitational forces is modelled as a constant force of magnitude 550 N.

(*a*) Show that $\sin\theta = \frac{1}{14}$.

(5)

When the car is travelling up the road at 16 m s^{-1}, the engine is switched off. The car comes to rest, without braking, having moved a distance y metres from the point where the engine was switched off. The resistance to motion from non-gravitational forces is again modelled as a constant force of magnitude 550 N.

(*b*) Find the value of y.

(4)

4.

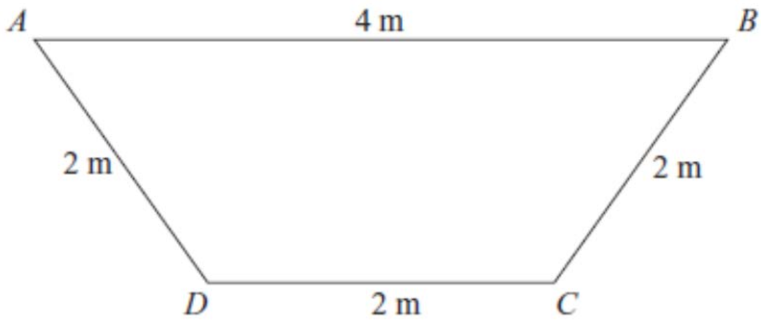


Figure 1

The trapezium *ABCD* is a uniform lamina with *AB* = 4 m and *BC* = *CD* = *DA* = 2 m, as shown in Figure 1.

(*a*) Show that the centre of mass of the lamina is $\frac{4\sqrt{3}}{9}$ m from *AB*.

(5)

The lamina is freely suspended from *D* and hangs in equilibrium.

(*b*) Find the angle between *DC* and the vertical through *D*.

(5)

5.

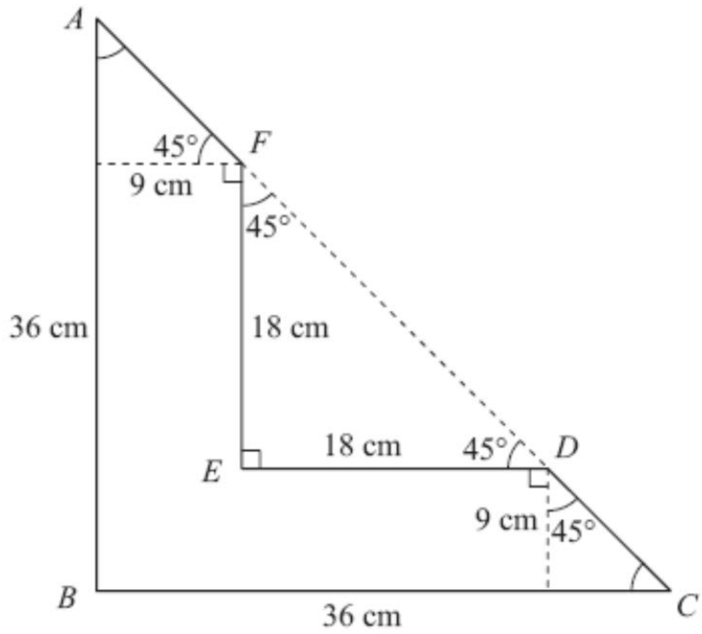


Figure 2

The uniform L-shaped lamina *ABCDEF*, shown in Figure 2, has sides *AB* and *FE* parallel, and sides *BC* and *ED* parallel. The pairs of parallel sides are 9 cm apart. The points *A*, *F*, *D* and *C* lie on a straight line.

$AB = BC = 36$ cm, $FE = ED = 18$ cm.

$\angle ABC = \angle FED = 90°$, and $\angle BCD = \angle EDF = \angle EFD = \angle BAC = 45°$.

(*a*) Find the distance of the centre of mass of the lamina from

(i) side *AB*,

(ii) side *BC*.

(7)

The lamina is freely suspended from *A* and hangs in equilibrium.

(*b*) Find, to the nearest degree, the size of the angle between *AB* and the vertical.

(3)

6.

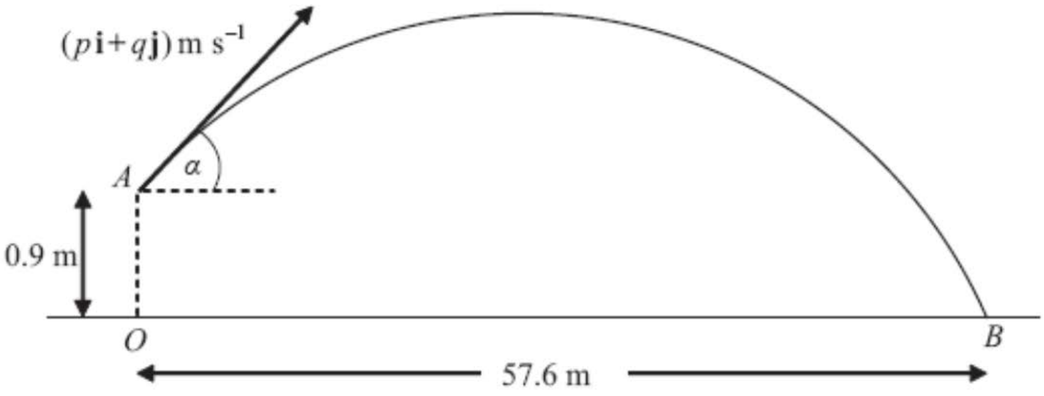


Figure 3

A cricket ball is hit from a point A with velocity of $(p\mathbf{i} + q\mathbf{j})$ m s^{-1}, at an angle α above the horizontal. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are respectively horizontal and vertically upwards. The point A is 0.9 m vertically above the point O, which is on horizontal ground.

The ball takes 3 seconds to travel from A to B, where B is on the ground and OB = 57.6 m, as shown in Figure 3. By modelling the motion of the cricket ball as that of a particle moving freely under gravity,

(*a*) find the value of p,

(2)

(*b*) show that q = 14.4,

(3)

(*c*) find the initial speed of the cricket ball,

(2)

(*d*) find the exact value of tan α.

(1)

(*e*) Find the length of time for which the cricket ball is at least 4 m above the ground.

(6)

(*f*) State an additional physical factor which may be taken into account in a refinement of the above model to make it more realistic.

(1)

7. A particle P of mass $3m$ is moving in a straight line with speed $2u$ on a smooth horizontal table. It collides directly with another particle Q of mass $2m$ which is moving with speed u in the opposite direction to P. The coefficient of restitution between P and Q is e.

(*a*) Show that the speed of Q immediately after the collision is $\frac{1}{5}(9e + 4)u$.

(5)

The speed of P immediately after the collision is $\frac{1}{2}u$.

(*b*) Show that $e = \frac{1}{4}$.

(4)

The collision between P and Q takes place at the point A. After the collision Q hits a smooth fixed vertical wall which is at right-angles to the direction of motion of Q. The distance from A to the wall is d.

(*c*) Show that P is a distance $\frac{3}{5}d$ from the wall at the instant when Q hits the wall.

(4)

Particle Q rebounds from the wall and moves so as to collide directly with particle P at the point B. Given that the coefficient of restitution between Q and the wall is $\frac{1}{5}$,

(*d*) find, in terms of d, the distance of the point B from the wall.

(4)

TOTAL FOR PAPER: 75 MARKS

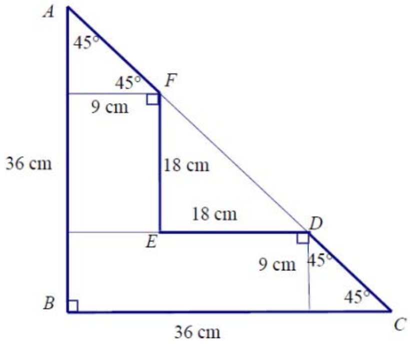
END

Question Number	Scheme	Marks
1.	Use of $m(\mathbf{v} - \boldsymbol{u}) = \boldsymbol{I}$ $0.1 \times (\mathbf{v} - 30\mathbf{i}) = -2\mathbf{i} - 4\mathbf{j}$ Solve for $\boldsymbol{v}$: $0.1\boldsymbol{v} = 3\mathbf{i} - 2\mathbf{i} - 4\mathbf{j} = \mathbf{i} - 4\mathbf{j}$ $\boldsymbol{v} = 10\mathbf{i} - 40\mathbf{j}$	M1 A1 M1 A1 **(4 marks)**
2. **(a)**	Speed = $\sqrt{8^2 + 48^2} = \sqrt{2368} = 48.7\ (\text{ms}^{-1})$	M1 A1 (2)
(b)	$\mathbf{a} = 2\mathbf{i} - 6t\,\mathbf{j}$ When $t = 4$, $\mathbf{a} = 2\mathbf{i} - 24\,\mathbf{j}\ (\text{ms}^{-2})$	M1 A1 A1 (3)
(c)	$\mathbf{r} = t^2\mathbf{i} - t^3\mathbf{j} + \mathbf{C}$ t = 1, $-4\mathbf{i} + \mathbf{j} = \mathbf{i} - \mathbf{j} + \mathbf{C}$, $\mathbf{C} = -5\mathbf{i} + 2\mathbf{j}$ $\mathbf{r} = (t^2 - 5)\mathbf{i} + \left(-t^3 + 2\right)\mathbf{j}$ When $t = 4$, $\mathbf{r} = (16 - 5)\mathbf{i} + (-64 + 2)\mathbf{j} = 11\mathbf{i} - 62\mathbf{j}$	M1 A1 M1 M1 A1 (5) **(10 marks)**
3. **(a)**	$20\,000 = 16F \quad (F = 1250)$ $\nearrow$ $F = 550 + 1000 \times 9.8 \sin\theta$ ft their F Leading to $\sin\theta = \frac{1}{14}$ $*$ cso	M1 A1 M1 A1ft A1 **(5)**
(b)	N2L $\nearrow$ $550 + 1000 \times 9.8 \times \sin\theta = 1000a$ $(550 + 1000 \times 9.8 \times \frac{1}{14} = 1000a$) or $1250 = 1000a$ $(a = (-)1.25)$ $v^2 = u^2 + 2as \Rightarrow 16^2 = 2 \times 1.25 \times y$ $y \approx 102$ accept 102.4, 100	 M1 A1 M1 A1 **(4)** **[9]**

Question Number	Scheme	Marks
4.		
(a)	For an appropriate division of the trapezium into standard shapes with:	
	correct ratio of masses	B1
	correct distances of c.o.m. from AB	B1
	e.g three equilateral triangles of height $\sqrt{3}$, mass *m* kg,	
	com $\frac{\sqrt{3}}{3}$ from bases of each	
	$3md = \left(m \times \frac{2}{3} \times \sqrt{3}\right) + \left(2 \times m \times \frac{1}{3}\sqrt{3}\right) = \frac{4\sqrt{3}}{3}m$,	M1 A1
	$d = \frac{4\sqrt{3}}{9}$ **AG**	A1
		(5)
(b)	Horizontal distance of c of m from D = 1m	B1
	Vertical distance $\sqrt{3} - \frac{4\sqrt{3}}{9} = \frac{5\sqrt{3}}{9}$ (0.962….)	B1
	$\tan^{-1}\frac{0.962...}{1}$	M1 A1ft
	Angle = 43.9°	A1
		(5)
		(10 marks)

Question Number	Scheme	Marks

5.



(a) Divide the shape into usable areas, e.g.:

Shape	C of mass	Units of mass
Rectangle 27 x 9	(13.5,4.5)	243 (6)
Right hand triangle	(30,3)	40.5 (1)
Top triangle	(3,30)	40.5 (1)
Rectangle 9 x 18	(4.5,18)	162 (4)

Mass ratios — **B1**
Centres of mass — **B1**

Take moments about AB: — **M1**
$6 \times 13.5 + 1 \times 30 + 4 \times 4.5 + 1 \times 3 = 132 = 12\bar{x}$ — **A(2,1,0)**

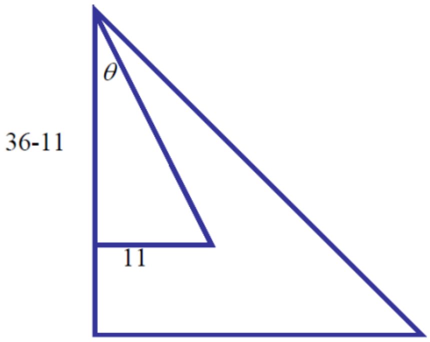
$\bar{x} = 11$(cm) solve for x (or y) co-ord — **A1**
$\bar{y} = 11$(cm) using the symmetry — **B1ft**

Alternative:

Shape	C of mass	Units of mass
Small triangle	(12,12)	.5 x 18x 18
Large triangle	(15,15)	.5 x 36 x 36

$\frac{1}{2} \times 36 \times 36 \times 12 - \frac{1}{2} \times 18 \times 18 \times 15 = \frac{1}{2} \times (36 \times 36 \times 18 \times 18)\ \bar{x}$ etc. — **(7)**

(b)



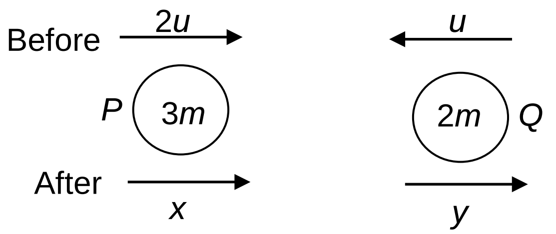
$\tan\theta = \frac{\bar{x}}{36-\bar{y}}$ — **M1**

$\tan\theta = \frac{11}{25} = 0.44$ — **A1ft**

$\theta = 24°$ — **A1**

(3)
[10]

Question Number	Scheme	Marks
6.		
(a)	Horizontal distance: $57.6 = p \times 3$	M1
	$p = 19.2$	A1
		(2)
(b)	Use $s = ut + \frac{1}{2}at^2$ for vertical displacement.	M1
	$-0.9 = q \times 3 - \frac{1}{2}g \times 3^2$	A1
	$-0.9 = 3q - \frac{9g}{2} = 3q - 44.1$	
	$q = \frac{43.2}{3} = 14.4$ ***AG***	A1 cso
		(3)
(c)	initial speed $\sqrt{p^2 + 14.4^2}$ (with their p)	M1
	$= \sqrt{576} = \underline{24}$ (m s^{-1})	A1 cao
		(2)
(d)	$\tan\alpha = \frac{14.4}{p} (= \frac{3}{4})$ (with their p)	B1
		(1)
(e)	When the ball is 4 m above ground:	
	$3.1 = ut + \frac{1}{2}at^2$ used	M1
	$3.1 = 14.4t - \frac{1}{2}gt^2$ o.e ($4.9t^2 - 14.4t + 3.1 = 0$)	A1
	$t = \frac{14.4 \pm \sqrt{(14.4)^2 - 4(4.9)(3.1)}}{2(4.9)}$ seen or implied	M1
	$t = \frac{14.4 \pm \sqrt{146.6}}{9.8}$ = 0.023389… or 2.70488… awrt 0.23 and 2.7	A1
	duration = 2.70488... − 0.23389...	M1
	= 2.47 or 2.5 (seconds)	A1
(f)	Eg. : Variable ‘g’, Air resistance, Speed of wind, Swing of ball, The ball is not a particle.	B1
		(1)
		[15]

Question Number	Scheme	Marks
7 (a)	 Correct use of NEL	M1*
	$y - x = e(2u + u)$ o.e.	A1
	CLM $(\rightarrow)$: $3m(2u) + 2m(-u) = 3m(x) + 2m(y)$ $(\square\ 4u = 3x + 2y)$	B1
	Hence x = y – 3eu, 4u = 3(y-3eu) + 2y, $(u(9e+4) = 5y)$	d*M1
	Hence, speed of $Q = \underline{\tfrac{1}{5}(9e+4)u}$ **AG**	A1 cso
		(5)
(b)	$x = y - 3eu = \dfrac{1}{5}(9e+4)u - 3eu$	M1[#]
	Hence, speed P $= \dfrac{1}{5}(4-6e)u = \dfrac{2u}{5}(2-3e)$ o.e.	A1
	$x = \dfrac{1}{2}u = \dfrac{2u}{5}(2-3e) \Rightarrow 5u = 8u - 12eu, \Rightarrow 12e = 3$ & solve for e	d[#]M1
	gives, $e = \tfrac{3}{12} \Rightarrow \underline{e = \tfrac{1}{4}}$ **AG**	A1
		(4)
(c)	Time taken by Q from A to the wall $= \underline{\dfrac{d}{y}} = \left\{\dfrac{4d}{5u}\right\}$	M1[†]
	Distance moved by P in this time $= \underline{\dfrac{u}{2}\times\dfrac{d}{y}}$ $\left(= \dfrac{u}{2}\left(\dfrac{4d}{5u}\right) = \dfrac{2}{5}d\right)$	A1
	Distance of P from wall $= d - x\left(\dfrac{d}{y}\right); = d - \tfrac{2}{5}d = \underline{\tfrac{3}{5}d}$ **AG**	d[†]M1; A1 cso
or		(4)
(c)	Ratio speed P:speed Q = x:y = $\dfrac{1}{2}u : \dfrac{1}{5}(\dfrac{9}{4}+4)u = \dfrac{1}{2}u : \dfrac{5}{4}u$ = 2:5	M1[†]
	So if Q moves a distance d, P will move a distance $\tfrac{2}{5}d$	A1
	Distance of P from wall $= d - \tfrac{2}{5}d; = \underline{\tfrac{3}{5}d}$ **AG** cso	d[†]M1; A1
		(4)

Examiner reports

Question 1

This question provided a very straight forward start to the paper, with many candidates scoring full marks. The majority of errors were due to slips in the arithmetic. Most candidates did consider a change in momentum, although the terms were sometimes subtracted in the wrong order. Candidates who added the initial and final momentum scored zero. The vectors caused confusion for some candidates who combined **i** and **j** terms inappropriately.

Too many candidates went on the find the magnitude of **v** which, although not penalised on this occasion, suggests that they were not reading the question properly, or did not know the difference between speed and velocity.

Question 2

Part (a): The majority of candidates were able to substitute $t = 4$ into the expression for **v** to find the velocity at $t = 4$. It seems that some candidates still fail to appreciate the difference between speed and velocity as several did not go on to find the speed.

Part (b): Very few candidates were unable to obtain **a** from **v** correctly. A significant number of candidates chose to go on to find the magnitude of the acceleration. This was not penalised on this occasion, but raises the question of whether they know that acceleration is a vector.

Part (c): This was often well answered, but the candidates did find it more difficult than parts (a) and (b). When the integration was completed correctly the constant was not always included, or was assumed to be $-4\mathbf{i} + \mathbf{j}$ without any working shown. When the constant was included, the substitution of $t = 1$ was a source of some careless errors. A common error was to see **i** and **j** combined so that $t^2\mathbf{i} - t^3\mathbf{j}$ became $1 - 1 = 0$. When the expression for the position vector was simplified there was often sign confusion (usually with the $-t^3\mathbf{j}$ term). There were also a significant number of careless arithmetic errors, with $16 - 5 = 9$ seen several times.

Despite having found an expression for **a** in terms of t in part (b), a small number of candidates failed to appreciate that integration was required for this solution and attempted to apply equations for constant acceleration.

Question 3

(a) This question was well answered with the great majority of candidates doing exactly what was required. The application of Newton's second law and resolution of the weight was usually correct and of course we always saw $\frac{1}{14}$. Candidates should be aware that in a question like this, where an answer is given, their working needs to demonstrate clearly how the answer is derived. Some candidates omitted essential working and did not convince us that they had reached the required answer. Others did not attempt to simplify their working, which they evaluated using their calculator, obtaining a decimal answer, which they then told us was approximately equal to $\frac{1}{14}$.

As usual, the more popular strategy in (b) seemed to be to use $F=ma$ and constant acceleration, rather than the work-energy principle. Some candidates repeated much of the working from part (a) to deduce that the total force acting parallel to the slope is 1250 N, others simply wrote it down as a straight forward deduction from the preceding work. For candidates who attempted to use the work-energy method, common errors included duplication of gravity or failure to include work done against resistance. Several candidates who used 550 N correctly in the first part went on to use 500 N in part (b) for no obvious reason.

Question 4

Part (a): Most candidates chose to split the shape into a composite body consisting of a rectangle and two triangles. The usual alternatives were a larger rectangle with two triangles removed, or a large triangle with a smaller one removed. The use of three equilateral triangles was also seen, but only rarely. The given answer was a help to candidates who were having difficulty dealing with the height of the trapezium and the positions of the centres of mass, prompting a review their work if necessary.

Initial results concerning the masses and positions of the centres of mass were often tabulated, thus making the work clearer, and easier for the examiner to check. Most candidates were able to take moments about a horizontal axis in order to find the required distance. Many took moments about *DC* rather than *AD*, but were able to go on to obtain the given result. The given answer did result in a few candidates producing contrived work to gain the correct answer. Almost without exception, all calculations were dealt with in surd form.

Part (b): There were many correct solutions, however many candidates chose to take moments about a vertical axis (often done in part (a)) to find the horizontal position of the centre of mass, completely failing to recognise the symmetry of the shape. Most were able to identify the correct triangle to use to find the required angle (often confirmed by referring back to the original diagram). Common errors included using a horizontal distance of 2 rather than 1 and slips in using the vertical distance given in part (a).

Question 5

In part (a) there were many entirely correct solutions to this question, with candidates employing a number of different strategies to split this shape into standard components. Most commonly this involved expressing it as the difference between two triangles, or splitting it into two rectangles and two triangles. Using just two triangles tended to produce the most concise and accurate solutions, although there was some confusion over the positions of the centres of mass of the triangles. Some candidates did not realise that they could work in terms of the horizontal and vertical distances from these vertices at *B* and *E* and went to considerable lengths to calculate the heights of the triangles measured from these vertices and then to use trigonometry. Candidates who divided the shape into four or more pieces frequently made errors in calculating the areas of these pieces or in locating their centres of mass. Another cause of errors was to double count a region, or even to leave it out entirely. A small number of candidates treated this as a structure made of rods rather than as a uniform lamina. A surprising number of candidates did not use the symmetry of the lamina to find the second distance, with many reworking a moments equation to get the same answer – or in some cases a different answer. Part (b) was very well answered by most candidates; the required angle was usually identified correctly and candidates could gain two marks for work clearly following from incorrect values in part (a). A common incorrect answer was 45°, from candidates who did not consider the geometry of the situation or use a diagram to help. Many candidates did not round their final answer to the nearest degree.

Question 6

Many candidates scored well in this question, with parts (a) to (d) generally answered correctly. However, a few candidates were confused right at the start with the vector form of the initial velocity and tried to bring resolution into the problem and so failed to find *p* and *q*.

Some candidates were initially confused about the direction of *p* and *q* – it was common to see attempts at parts (a) and (b) relabelled when a candidate discovered their error. In (b) there was often insufficient evidence that the given answer had been reached correctly – essential steps in the working were omitted. Some candidates used long drawn out trigonometric methods to find tan α in part (d), often finding cos α and sin α before finally reaching $\frac{3}{4}$.

In part (e) most candidates used $s = ut + \frac{1}{2}at^2$ from the point of projection but there were a number of other possible methods which were also successful. It was evident that some candidates are relying on the use of calculators to solve quadratic equations. When the initial quadratic equation was incorrect, marks were often lost as a result of failing to show sufficient evidence of use of an appropriate method. It was common to see 4 used in place of 3.1 in the initial equation. Candidates using alternative approaches often got lost in the complexities of the logic of what they were trying to do.

The responses in part (f) showed that virtually all candidates could find (at least) one physical factor which could also be taken into account although a few miss-worded their answers to imply the opposite. For example, many suggested “air resistance”, but it was also common to see “no air resistance”.

Question 7

Candidates made a confident start to this question, but in parts (a) and (b), poor algebraic skills and the lack of a clear diagram with the directions marked on it hampered weaker candidates' attempts to set up correct and consistent (or even physically possible) equations. The direction of P after impact was not given and those candidates who took its direction as reversed ran into problems when finding the value of e. Many realised that they had chosen the wrong direction and went on to answer part (b) correctly but some did not give an adequate explanation for a change of sign for their velocity of P. Algebraic and sign errors were common, and not helped by candidates' determination to reach the given answers.

Parts (c) and (d) caused the most problems. They could be answered using a wide variety of methods, some more formal than others. Many good solutions were seen but unclear reasoning and methods marred several attempts. Too many solutions were sloppy, with u or d appearing and disappearing through the working. A few words describing what was being calculated or expressed at each stage would have helped the clarity of solutions greatly. Students need to be reminded yet again that all necessary steps need to be shown when reaching a given answer. Too many simply stated the answer $\frac{3d}{5}$ without the explanation to support it.

Statistics for M2 Practice Paper Bronze 4

				Mean average scored by candidates achieving grade:							
Qu	**Max Score**	**Modal score**	**Mean %**	**ALL**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
1	4		93.8	3.75	3.94	3.84	3.59	3.44	3.23	3.44	2.12
2	10		89.1	8.91	9.75	9.39	8.59	7.94	7.06	6.42	3.13
3	9		83.3	7.50		8.29	7.44	6.07	4.57	2.99	0.89
4	10		78.4	7.84	9.47	8.61	6.48	5.32	3.95	3.66	2.06
5	10		74.5	7.45	8.78	8.35	6.49	5.63	3.72	1.59	1.61
6	15		75.5	11.33		12.33	9.81	7.69	5.32	4.10	2.90
7	17		71.6	12.18		13.60	9.42	7.79	5.57	5.92	1.12
	75		**78.6**	**58.96**		**64.41**	**51.82**	**43.88**	**33.42**	**28.12**	**13.83**