Paper Reference(s)

6678/01

Edexcel GCE

Mechanics M2

Bronze Level B2

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
74	68	61	54	46	38

1. [*In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane*.]

A particle P moves in such a way that its velocity $\mathbf{v}$ m s^{-1} at time t seconds is given by

$$\mathbf{v} = (3t^2 - 1)\mathbf{i} + (4t - t^2)\mathbf{j}.$$

(*a*) Find the magnitude of the acceleration of P when $t = 1$.

(5)

Given that, when $t = 0$, the position vector of P is $\mathbf{i}$ metres,

(*b*) find the position vector of P when $t = 3$.

(5)

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2. A lorry of mass 1800 kg travels along a straight horizontal road. The lorry's engine is working at a constant rate of 30 kW. When the lorry's speed is 20 m s^{-1}, its acceleration is 0.4 m s^{-2}. The magnitude of the resistance to the motion of the lorry is R newtons.

(*a*) Find the value of R.

(4)

The lorry now travels up a straight road which is inclined at an angle α to the horizontal, where $\sin \alpha = \frac{1}{12}$. The magnitude of the non-gravitational resistance to motion is R newtons. The lorry travels at a constant speed of 20 m s^{-1}.

(*b*) Find the new rate of working of the lorry's engine.

(5)

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3. A ball of mass 0.5 kg is moving with velocity 12**i** m s^{-1} when it is struck by a bat. The impulse received by the ball is (–4**i** + 7**j**) N s. By modelling the ball as a particle, find

(*a*) the speed of the ball immediately after the impact,

(4)

(*b*) the angle, in degrees, between the velocity of the ball immediately after the impact and the vector **i**,

(2)

(*c*) the kinetic energy gained by the ball as a result of the impact.

(2)

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4. At time t seconds the velocity of a particle P is $[(4t - 5)\mathbf{i} + 3\mathbf{j}]$ m s^{-1}. When $t = 0$, the position vector of P is $(2\mathbf{i} + 5\mathbf{j})$ m, relative to a fixed origin O.

(*a*) Find the value of t when the velocity of P is parallel to the vector $\mathbf{j}$.

(1)

(*b*) Find an expression for the position vector of P at time t seconds.

(4)

A second particle Q moves with constant velocity $(-2\mathbf{i} + c\mathbf{j})$ m s^{-1}. When $t = 0$, the position vector of Q is $(11\mathbf{i} + 2\mathbf{j})$ m. The particles P and Q collide at the point with position vector $(d\mathbf{i} + 14\mathbf{j})$ m.

(*c*) Find

(i) the value of c,

(ii) the value of d.

(5)

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5. [*In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

A ball of mass 0.5 kg is moving with velocity $(10\mathbf{i} + 24\mathbf{j})$ m s^{-1} when it is struck by a bat. Immediately after the impact the ball is moving with velocity $20\mathbf{i}$ m s^{-1}.

Find

(*a*) the magnitude of the impulse of the bat on the ball,

(4)

(*b*) the size of the angle between the vector $\mathbf{i}$ and the impulse exerted by the bat on the ball,

(2)

(*c*) the kinetic energy lost by the ball in the impact.

(3)

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6. Three identical particles, A, B and C, lie at rest in a straight line on a smooth horizontal table with B between A and C. The mass of each particle is m. Particle A is projected towards B with speed u and collides directly with B. The coefficient of restitution between each pair of particles is $\frac{2}{3}$.

(*a*) Find, in terms of u,

(i) the speed of A after this collision,

(ii) the speed of B after this collision.

(7)

(*b*) Show that the kinetic energy lost in this collision is $\frac{5}{36}mu^2$.

(4)

After the collision between A and B, particle B collides directly with C.

(*c*) Find, in terms of u, the speed of C immediately after this collision between B and C.

(4)

7.

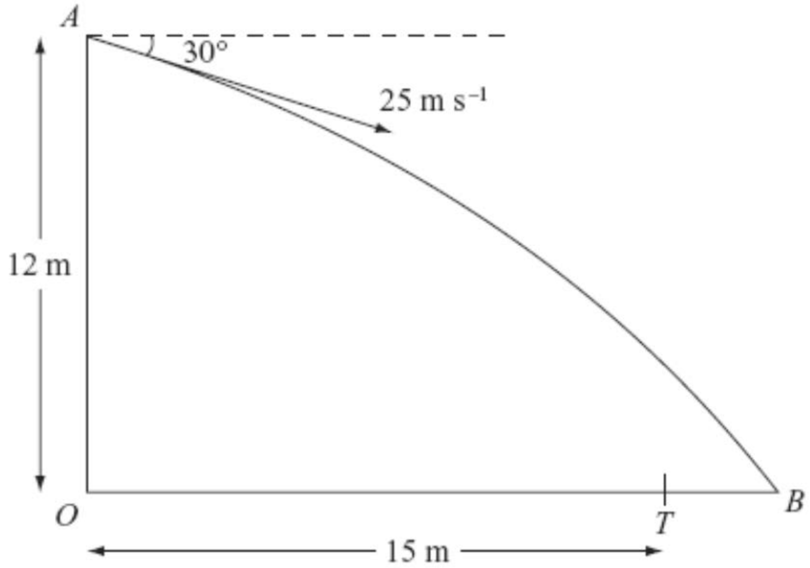


Figure 4

A ball is thrown from a point A at a target, which is on horizontal ground. The point A is 12 m above the point O on the ground. The ball is thrown from A with speed 25 m s^{-1} at an angle of 30° below the horizontal. The ball is modelled as a particle and the target as a point T. The distance OT is 15 m. The ball misses the target and hits the ground at the point B, where OTB is a straight line, as shown in Figure 4. Find

(*a*) the time taken by the ball to travel from A to B,

(5)

(*b*) the distance TB.

(4)

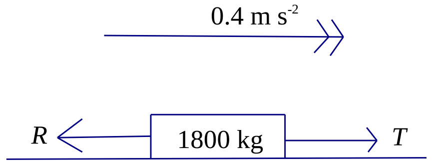
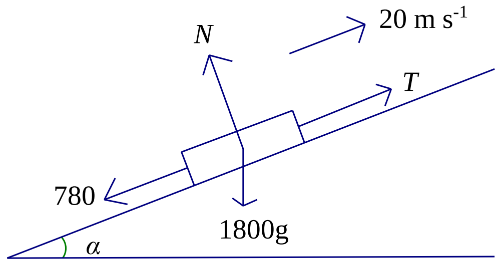
The point X is on the path of the ball vertically above T.

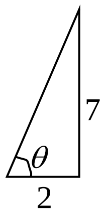
(*c*) Find the speed of the ball at X.

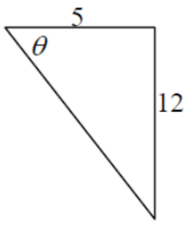
(5)

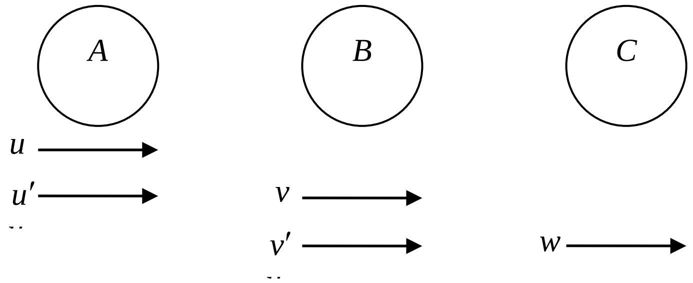
TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1. (a)	$\mathbf{a} = \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = 6t\mathbf{i} + (4-2t)\mathbf{j}$	M1 A1
	When $t = 1$, $\mathbf{a} = 6\mathbf{i} + 2\mathbf{j}$	DM1
	$\lvert\mathbf{a}\rvert = \sqrt{6^2 + 2^2} = \sqrt{40} = 6.32\ (\mathrm{m\ s^{-2}})$	DM1 A1 (5)
(b)	$\mathbf{r} = \int (3t^2 - 1)\mathbf{i} + (4t - t^2)\mathbf{j}\ \mathrm{d}t$	M1
	$= (t^3 - t + C)\mathbf{i} + (2t^2 - \frac{1}{3}t^3 + D)\mathbf{j}$	A1
	$t = 0, \mathbf{r} = \mathbf{i} \Rightarrow C = 1, D = 0$	DM1
	When $t = 3, \mathbf{r} = 25\mathbf{i} + 9\mathbf{j}$ (m)	DM1 A1 (5) **10**
2 (a)		B1
	$T = \frac{30000}{20}\ \ (=1500)$	M1
	$T - R = 1800a$	A1
	$T - R = 1800 \times 0.4$	
	$R = 1500 - 1800 \times 0.4$	A1
	$= 780$	
(b)		M1
	$T - 1800g\sin\alpha - R = 0$	A1
	$T = 1800 \times \frac{1}{12}g + 780$	M1
	Power $= \left(1800 \times \frac{1}{12}g + 780\right) \times 20$	A1
	$= 45000$ W or 45 kW	A1

Question Number	Scheme	Marks
3. (a)	$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ $-4\mathbf{i} + 7\mathbf{j} = 0.5(\mathbf{v} - 12\mathbf{i})$	M1
	$4\mathbf{i} + 14\mathbf{j} = \mathbf{v}$	A1
	Speed $= \sqrt{16+196} = \sqrt{212}$ m s^{-1} (14.6 or better)	M1 A1
		(4)
(b)		
	$\tan\theta = \frac{7}{2}$	M1
	$\theta = 74.0....$ $\theta = 74°$	A1ft
		(2)
(c)	Gain in K.E. $= \frac{1}{2} \times 0.5(212 - 12^2)$, $= 17$ J	M1 A1
		(2) **8**
4. (a)	$t = \frac{5}{4}$	B1
(b)	$\mathbf{r} = (2t^2 - 5t)\mathbf{i} + 3t\mathbf{j}(+\mathbf{c})$	M1
	$t = 0 \quad 2\mathbf{i} + 5\mathbf{j} = \mathbf{c}$	A1
	$\mathbf{r} = (2t^2 - 5t)\mathbf{i} + 3t\mathbf{j} + (2\mathbf{i} + 5\mathbf{j})$	M1
	$(2t^2 - 5t + 2)\mathbf{i} + (3t + 5)\mathbf{j}$	A1
(c)	$\mathbf{r}_Q = 11\mathbf{i} + 2\mathbf{j} - 2t\mathbf{i} + ct\mathbf{j}$	B1
	$(11 - 2t)\mathbf{i} + (2 + ct)\mathbf{j}$	
	$\mathbf{r}_P = (2t^2 - 5t + 2)\mathbf{i} + (3t + 5)\mathbf{j}$	
	$\mathbf{r}_Q = \mathbf{r}_P = d\mathbf{i} + 14\mathbf{j}$	
	$3t + 5 = 14$ | $2t^2 - 3t - 9$ $(2t + 3)(t - 3) = 0$ $t = 3$	M1
	$t = 3$ | A1 ft	A1
	$2 + ct = 14 \Rightarrow c = 4$	A1 ft
	$d = 11 - 2 \times 3 = 5$ or $d = 2 \times 3^2 - 5 \times 3 + 2 \Rightarrow d = 5$	

Question Number	Scheme	Marks
5. (a)	$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$	
	$= 0.5 \times 20\mathbf{i} - 0.5(10\mathbf{i} + 24\mathbf{j})$	**M1**
	$= 5\mathbf{i} - 12\mathbf{j}$	**A1**
	$\lvert 5\mathbf{i} - 12\mathbf{j} \rvert = 13$ Ns	**M1 A1**
		(4)
(b)		
	$\tan\theta = \frac{12}{5}$	**M1**
	$\theta = 67.38$	
	$\theta = 67.4°$	**A1**
		(2)
(c)		**M1 A1**
	K.E.lost = $\frac{1}{2} \times 0.5(10^2 + 24^2) - \frac{1}{2} \times 0.5 \times 20^2$	
		A1
	$= 69$ J	**(3)**
		[9]

Question Number	Scheme	Marks
6. (a)		
	Momentum: $u = u' + v$	M1 A1
	NEL: $v - u' = eu$	M1 A1
	$2v = u(1 + \frac{2}{3}),\ v = \frac{1}{2}u \times \frac{5}{3} = \frac{5u}{6}$	M1 A1
	$u' = u - v = \frac{u}{6}$	A1
		(7)
(b)	KE lost = $\frac{1}{2}mu^2 - \left(\frac{1}{2}m \times \frac{25}{36}u^2 + \frac{1}{2}m \times \frac{1}{36}u^2\right)$ their speeds	M1
	$= \frac{1}{2}mu^2 - \left(\frac{1}{2}m \times \frac{26}{36}u^2\right)$	A2 – 1ee
	$= \frac{1}{2}mu^2 \times \frac{10}{35} = \frac{5}{36}mu^2$ **AG**	A1
		(4)
(c)	Speed of C = $\frac{1}{2}\left(\frac{1}{2}u\left(\frac{5}{3}\right)\right)\left(\frac{5}{3}\right) = \frac{1}{2}\cdot\frac{5u}{6}\cdot\frac{5}{3}, = \frac{25}{36}u$	M1 A1 M1 A1
		(4) **(15 marks)**

Question Number	Scheme		Marks
7. (a)	$(\downarrow)$ $u_y = 25\sin 30° \ (=12.5)$		B1
	$12 = 12.5t + 4.9t^2$	−1 each error	M1 A2, 1, 0
	Leading to $t = 0.743$, 0.74		A1 (5)
(b)	$(\rightarrow)$ $u_x = 25\cos 30° \left(= \frac{25\sqrt{3}}{2} \approx 21.65\right)$		B1
	$OB = 25\cos 30° \times t \ (\approx 16.094\,58)$	ft their (a)	M1 A1ft
	$TB \approx 1.1$ (m)	awrt 1.09	A1 (4)
(c)	$(\rightarrow)$ $15 = u_x \times t \Rightarrow t = \frac{15}{u_x} (= \frac{2\sqrt{3}}{5} \approx 0.693$ or $0.69)$		M1 A1
	either $(\downarrow)$ $v_y = 12.5 + 9.8t \ (\approx 19.2896)$		M1
	$V^2 = u_x^{\ 2} + v_y^{\ 2} \ (\approx 840.840)$		
	$V \approx 29 \ (\text{ms}^{-1})$, 29.0		M1 A1 (5)
			(14 marks)

Examiner reports

Question 1

Most candidates gave confident responses to this question, which is of a similar style to many that they will have seen before. There are still a few candidates who do not understand that it is not appropriate to use the *suvat* equations in a question involving variable acceleration.

In part (a) the differentiation was usually correct. There were a few arithmetic errors in substituting $t = 1$, and some candidates left their answer for the acceleration as a vector rather than going on to find the magnitude.

In part (b) the integration was often correct. Some candidates did not have a constant of integration (or use a definite integral), and some were confused about the value of t to use; use of $t = 1$ was a common error. Of those candidates who completed the integration correctly, there were several who made arithmetic slips in substituting $t = 3$.

Question 2

In Q2(a) candidates demonstrated a good understanding of the concept of power and many correct solutions were seen, with only a small number of candidates making errors in the equation of motion.

In Q2(b) many correct solutions were seen, but there were some errors in setting up the equation of motion. A few candidates did not include g in their weight component. There were also a significant number who, having correctly found R in Q2(a), then used $R = 720$ in Q2(b). Often it was clear that this was an error and a case of mis-copying from Q2(a).

Question 3

Many candidates would have gained more marks in this question if they had checked that they had actually found the quantities asked for in the question. Despite some arithmetic errors in dividing by 0.5, many candidates successfully found the velocity after impact but only about half of those went on the find the speed asked for in the question.

In part (b), most candidates managed to find an appropriate angle, although there was some confusion over whether to use the velocity after impact or the impulse. Sometimes the fraction for $\tan\theta$ was the wrong way up, and occasionally sine or cosine were used, but often incorrectly. Those who had sign errors in their **v** often failed to realise that an obtuse angle would then be required.

Finding the change in kinetic energy in part (c)caused difficulties for those candidates who did not realise that energy is a scalar quantity and that the v^2 required was merely the square of the speed found in (a). Some candidates had accuracy errors due to the use of a rounded value for v^2. Some only found the final kinetic energy rather than the change in kinetic energy.

Question 4

Q4(a) was usually correct, but a minority of candidates did not realise that if the particle is moving parallel to vector **j** then the **i** component of the velocity must be zero. There were also a number of errors in solving $4t - 5 = 0$.

In Q4(b) some candidates did not attempt to integrate the velocity vector, and some did not have a constant of integration, but there were many correct solutions to this part of the question. Candidates who attempted to use the *suvat* equations usually went wrong because they assumed that the velocity was constant.

Q4(c) proved to be more challenging. There were many neat and concise solutions, but some candidates were not able to use the information given to set up equations to find values for *c* and *d*; they did not seem to realise that for the two particles to collide their position vectors needed to be the same.

Question 5

This question was well attempted by a majority of candidates.

In part (a) the most common incorrect answer was a sign error leading to an impulse of (5**i** + 12**j**) Ns rather than (5**i** – 12**j**). Some candidates failed to apply the impulse formula correctly, adding momentum rather than subtracting. Many students forgot to calculate the magnitude of their impulse. A few candidates started by finding the initial and final speeds of the ball and ignored the two dimensional nature of the problem, never producing a vector equation for the impulse or appropriate work using trigonometry.

In part (b) a common error was to find the angle for the initial velocity rather than the impulse. A minority of candidates were confused over which angle was required or made a trigonometric error, using the ratio $\frac{5}{12}$ rather than $\frac{12}{5}$ to find the angle.

For part (c) although there were many completely correct solutions, some candidates were unable to cope with using vectors to find speed and hence Kinetic Energy. The most common errors were, for example, to find $\sqrt{10^2 + 24^2}$ and then forget to square it or to attempt to square the vector velocity, treating $(10\mathbf{i} + 24\mathbf{j})^2$ as an algebraic expression and retaining **i** and **j** components in the answer.

Question 6

Part (a) Candidates were confident in producing equations for conservation of momentum and the impact law. Some candidates were inconsistent in the uses of signs in the two equations, and on occasions produced some dubious algebra which resulted in either fortuitously ‘correct’ results or the speeds in ‘reverse’ order. Candidates should be encouraged to look at their results in the context of the question – it is impossible for particle *A* to be travelling in the same direction as *B*, but more quickly, after the impact detailed in the question. This question asked for the speeds of the two particles after the collision, and some candidates are still not making the distinction between speed and velocity.

Part (b): Provided the answers to part (a) had been obtained correctly, most candidates were able to achieve the given answer. Most tackled (final KE – initial KE) successfully for the whole system. The few sign errors that were made came in solutions in which energy loss was treated separately for each ball. A few made algebra errors but most got to the given answer with ease and even managed to justify losing a minus sign at the end where necessary.

Part (c): Very few candidates took the obvious short cut (answer from part (a) multiplied by $\frac{5}{6}$). Everyone else laboured through the simultaneous equations. A handful got lost or confused but the vast majority worked through successfully as they had in part (a).

Question 7

This was a well answered question with almost all candidates working in appropriate vertical and horizontal directions. However, some candidates were confused by the fact that the ball was projected below the horizontal – many of these assuming that it was projected horizontally and then followed the path illustrated.

(a) Most candidates used $s=ut + ½ at^2$ to produce a quadratic equation albeit often with a sign error. When marks were lost this was usually because of difficulties with positive and negative vertical terms. The alternative, longer method, of finding *v* then *t* was not infrequent.

(b) The method was clearly understood but the numbers involved were very sensitive to rounding errors. Too many candidates over-rounded their value for *t* and obtained the inaccurate value of 1.02.

(c) It was pleasing to see that most candidates adopted the correct approach of finding *t* then v_y but some failed to then combine v_x and v_y to obtain the speed. Again, sign errors with the acceleration resulted in errors in v_y being less than the original vertical component, which one would have hoped might ring an alarm bell.

Few candidates used the energy method and those that did often used the incorrect vertical displacement.

Statistics for M2 Practice Paper Bronze 2

				Mean average scored by candidates achieving grade:							
Qu	**Max Score**	**Modal score**	**Mean %**	**ALL**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
1	10		91.5	9.15	9.80	9.67	9.42	8.99	8.38	7.46	5.33
2	9	9	89.0	8.04	8.71	8.43	7.79	7.10	6.19	4.87	3.28
3	8		80.3	6.42	7.45	7.04	6.37	5.83	5.12	4.31	3.16
4	10	10	81.0	8.08	9.65	9.04	7.45	5.76	4.87	4.11	1.80
5	9		78.4	7.06	8.34	7.83	7.04	6.47	5.61	4.75	3.25
6	15		86.7	13.00	14.59	14.02	12.23	10.66	6.86	6.07	2.12
7	14		73.4	10.28		12.33	10.33	9.02	7.42	5.93	3.84
	75		**82.7**	**62.03**		**68.36**	**60.63**	**53.83**	**44.45**	**37.50**	**22.78**