GCE Examinations

Mechanics
Module M1

Advanced Subsidiary / Advanced Level

Paper J

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

When a numerical value of g is required, use $g = 9.8 \text{ m s}^{-2}$.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Shaun Armstrong & Chris Huffer

1. At time $t = 0$, a particle of mass 2 kg has velocity $(8\mathbf{i} + \lambda\mathbf{j})$ m s^{-1} where $\mathbf{i}$ and $\mathbf{j}$ are horizontal perpendicular unit vectors and $\lambda > 0$.

Given that the speed of the particle at time $t = 0$ is 17 m s^{-1},

(a) find the value of λ. **(3 marks)**

The particle experiences a constant retarding force $\mathbf{F}$ so that when $t = 5$, it has velocity $(3\mathbf{i} + 5\mathbf{j})$ m s^{-1}.

(b) Show that $\mathbf{F}$ can be written in the form $\mu(\mathbf{i} + 2\mathbf{j})$ N where μ is a constant which you should find.
(5 marks)

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2. A monk uses a small brush to clean the stone floor of a monastery by pushing the brush with a force of P Newtons at an angle of 60° to the vertical. He moves the brush at a constant speed. The mass of the brush is 0.5 kg and the coefficient of friction between the brush and the floor is $\frac{1}{\sqrt{3}}$. The brush is modelled as a particle and air resistance is ignored.

(a) Show that $P = \frac{g}{2}$ Newtons. **(7 marks)**

(b) Explain why it is reasonable to ignore air resistance in this situation. **(1 mark)**

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3. A small van of mass 1500 kg is used to tow a car of mass 750 kg by means of a rope of length 9 m joined to both vehicles. The van sets off with the rope slack and reaches a speed of 2 m s^{-1} just before the rope becomes taut and jerks the car into motion. Immediately after the rope becomes taut, the van and car travel with common speed V m s^{-1}.

(a) Show that $V = \frac{4}{3}$. **(3 marks)**

(b) Calculate the magnitude of the impulse on the car when the rope tightens. **(2 marks)**

The van and car eventually reach a steady speed of 18 m s^{-1} with the rope taut when a child runs out into the road, 30 m in front of the van. The van driver brakes sharply and decelerates uniformly to rest in a distance of 27 m.

It takes the driver of the car 1 second to react to the van starting to brake. He then brakes and the car decelerates uniformly at f m s^{-2}, coming to rest before colliding with the van.

(c) Find the set of possible values of f. **(5 marks)**

4.

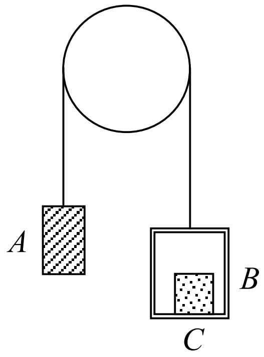

Fig. 1

Figure 1 shows a weight A of mass 6 kg connected by a light, inextensible string which passes over a smooth, fixed pulley to a box B of mass 5 kg. There is an object C of mass 3 kg resting on the horizontal floor of box B.

The system is released from rest. Find, giving your answers in terms of g,

(a) the acceleration of the system, **(4 marks)**

(b) the force on the pulley. **(3 marks)**

(c) Show that the reaction between C and the floor of B is $\frac{18}{7}g$ newtons. **(3 marks)**

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5. Two flies P and Q, are crawling vertically up a wall. At time $t = 0$, the flies are at the same height above the ground, with P crawling at a steady speed of 4 cm s^{-1}.

Q starts from rest at time $t = 0$ and accelerates uniformly to a speed of 6 cm s^{-1} in 6 seconds. Fly Q then maintains this speed.

(a) Find the value of t when the two flies are moving at the same speed. **(3 marks)**

(b) Sketch on the same diagram, speed-time graphs to illustrate the motion of the two flies.

(3 marks)

Given that the distance of the two flies from the top of the wall at time $t = 0$ is x cm and that Q reaches the top of the wall first,

(c) show that $x > 36$. **(5 marks)**

Turn over

6.

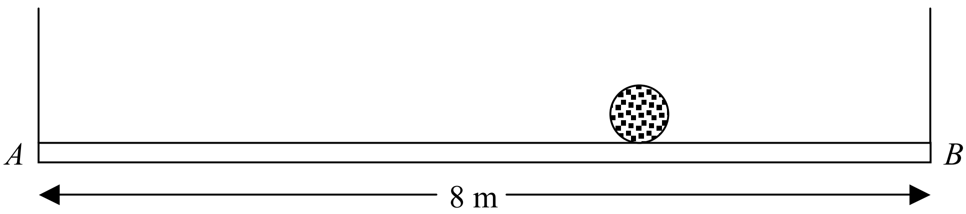

Fig. 2

Figure 2 shows a uniform plank *AB* of length 8 m and mass 50 kg suspended horizontally by two light vertical inextensible strings attached at either end of the plank. The maximum tension that either string can support is $40g$ N.

A rock of mass M kg is placed on the plank at A and rolled along the plank to B without either string breaking.

(a) Explain, with the aid of a sketch-graph, how the tension in the string at A varies with x, the distance of the rock from A.

(3 marks)

(b) Show that $M \leq 15$. **(5 marks)**

The first rock is removed and a second rock of mass 20 kg is placed on the plank.

(c) Find the fraction of the plank on which the rock can be placed without one of the strings breaking.

(6 marks)

7. At 6 a.m. a cargo ship has position vector $(7\mathbf{i} + 56\mathbf{j})$ km relative to a fixed origin O on the coast and moves with constant velocity $(9\mathbf{i} - 6\mathbf{j})$ km h^{-1}.

A ferry sails from O at 6 a.m. and moves with constant velocity $(12\mathbf{i} + 18\mathbf{j})$ km h^{-1}. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed due east and due north respectively.

(a) Show that the position vector of the cargo ship t hours after 6 a.m. is given by

$$[(7 + 9t)\mathbf{i} + (56 - 6t)\mathbf{j}] \text{ km},$$

and find the position vector of the ferry in terms of t. **(3 marks)**

(b) Show that if both vessels maintain their course and speed, they will collide and find the time and position vector at which this occurs.

(6 marks)

At 8 a.m. the captain of the ferry realises that a collision is imminent and changes course so that the ferry now has velocity $(21\mathbf{i} + 6\mathbf{j})$ km h^{-1}.

(c) Find the distance between the two ships at the time when they would have collided.

(5 marks)

END