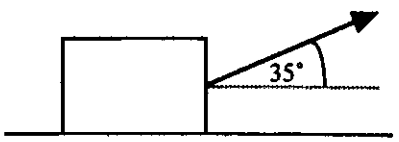
Take $g = 9{\cdot}8$ ms$^{-2}$ and give all answers correct to 3 significant figures where necessary.

1. Briefly define the following terms used in modelling in Mechanics:
   (i) lamina, (ii) uniform rod, (iii) smooth surface, (iv) particle. **(4 marks)**

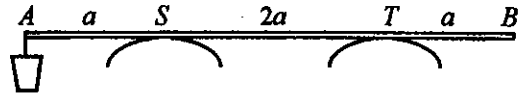
2. Two forces **F** and **G** are given by $\mathbf{F} = (6\mathbf{i} - 5\mathbf{j})$ N, $\mathbf{G} = (3\mathbf{i} + 17\mathbf{j})$ N, where **i** and **j** are unit vectors in the $x$ and $y$ directions respectively and the unit of length on each axis is 1 cm.
   (a) Find the magnitude of **R**, the resultant of **F** and **G**. **(3 marks)**
   (b) Find the angle between the direction of **R** and the positive $x$-axis. **(2 marks)**
   **R** acts through the point $P(-4, 3)$. $O$ is the origin $(0, 0)$.
   (c) Use the fact that $OP$ is perpendicular to the line of action of **R** to calculate the moment of **R** about an axis through the origin and perpendicular to the $x$-$y$ plane. **(3 marks)**

3. A string is attached to a packing case of mass 12 kg, which is at rest on a rough horizontal plane. When a force of magnitude 50 N is applied at the other end of the string, and the string makes an angle of 35° with the vertical as shown, the case is on the point of moving.




   (a) Find the coefficient of friction between the case and the plane. **(5 marks)**
   The force is now increased, with the string at the same angle, and the case starts to move along the plane with constant acceleration, reaching a speed of 2 ms$^{-1}$ after 4 seconds.
   (b) Find the magnitude of the new force. **(5 marks)**
   (c) State any modelling assumptions you have made about the case and the string. **(2 marks)**

4. A uniform yoke $AB$, of mass 4 kg and length $4a$ m, rests on the shoulders $S$ and $T$ of two oxen.




   $AS = TB = a$ m. A bucket of mass $x$ kg is suspended from $A$.
   (a) Show that the vertical force on the yoke at $T$ has magnitude $(2 - \frac{1}{2}x)g$ N and find, in terms of $x$ and $g$, the vertical force on the yoke at $S$. **(7 marks)**
   (b) If the ratio of these vertical forces is 5 : 1, find the value of $x$. **(3 marks)**
   (c) Find the maximum value of $x$ for which the yoke will remain horizontal. **(2 marks)**

5. Two small smooth spheres $A$ and $B$, of equal radius but masses $m$ kg and $km$ kg respectively, where $k > 1$, move towards each other along a straight line and collide directly. Immediately before the collision, $A$ has speed 5 m s$^{-1}$ and $B$ has speed 3 m s$^{-1}$. In the collision, the impulse exerted by $A$ on $B$ has magnitude $7km$ Ns.

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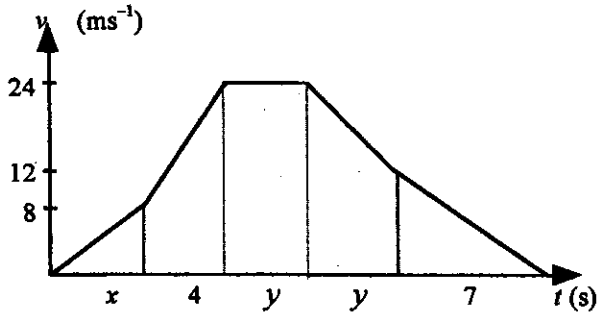
5. continued ...

(a) Find the speed of $B$ after the impact. **(3 marks)**

(b) Show that the speed of $A$ immediately after the collision is $(7k - 5)$ ms$^{-1}$ and deduce that the direction of $A$'s motion is reversed. **(5 marks)**

$B$ is now given a further impulse of magnitude $mu$ Ns, as a result of which a second collision between it and $A$ occurs.

(c) Show that $u > k(7k - 1)$. **(4 marks)**

6. 


The velocity-time graph illustrates the motion of a particle which accelerates from rest to 8 ms$^{-1}$ in $x$ seconds and then to 24 ms$^{-1}$ in a further 4 seconds. It then travels at a constant speed for another $y$ seconds before decelerating to 12 ms$^{-1}$ over the next $y$ seconds and then to rest in the final 7 seconds of its motion.

Given that the total distance travelled by the particle is 496 m,

(a) show that $2x + 21y = 195$. **(4 marks)**

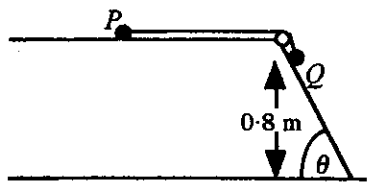
Given also that the average speed of the particle during its motion is 15·5 ms$^{-1}$,

(b) show that $x + 2y = 21$. **(3 marks)**

(c) Hence find the values of $x$ and $y$, **(3 marks)**

(d) Write down the acceleration for each section of the motion. **(3 marks)**

7. Two particles $P$ and $Q$, of masses $2m$ and $3m$ respectively, are connected by a light string. Initially, $P$ is at rest on a smooth horizontal table. The string passes over a small smooth pulley and $Q$ rests on a rough plane inclined at an angle $\theta$ to the horizontal, where $\tan\theta = \frac{4}{3}$. The coefficient of friction between $Q$ and the inclined plane is $\frac{1}{6}$.




The system is released from rest with $Q$ at a distance of 0·8 metres above a horizontal floor.

(a) Show that the acceleration of $P$ and $Q$ is $\frac{21g}{50}$, stating a modelling assumption which you must make to ensure that both particles have the same acceleration. **(7 marks)**

(b) Find the speed with which $Q$ hits the floor. **(2 marks)**

After $Q$ hits the floor and does not rebound, $P$ moves a further 0·2 m until it hits the pulley.

(c) Find the total time after the system is released before $P$ hits the pulley. **(5 marks)**