Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. A particle P, of mass 2·5 kg, initially at rest at the point O, moves on a smooth horizontal surface with constant acceleration $(\mathbf{i} + 2\mathbf{j})$ ms^{-2}, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the directions due East and due North respectively. Find

   (a) the velocity vector of P at time t seconds after it leaves O, **(2 marks)**

   (b) the magnitude and direction of the velocity of P when $t = 7$, **(3 marks)**

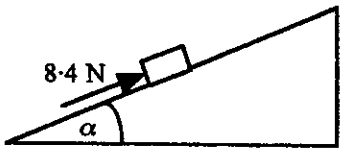
   (c) the magnitude, in N, of the force acting on P. **(2 marks)**

2. An iron bar AB, of length 4 m, is kept in a horizontal position by a support at A and a wire attached to the point P on the bar, where $PB = 0{\cdot}85$ m. The bar is modelled as a non-uniform rod whose centre of mass is at G, where $AG = 1{\cdot}4$ m, and the wire is modelled as a light inextensible string. Given that the tension in the wire is 12 N, calculate

   (a) the weight of the bar, **(4 marks)**

   (b) the magnitude of the reaction on the bar at A. **(2 marks)**

   (c) State briefly how you have used the given modelling assumption about the bar. **(1 mark)**

3. 

   A small packet, of mass 1·2 kg, is at rest on a rough plane inclined at an angle α to the horizontal. The coefficient of friction between the packet and the plane is $\frac{1}{8}$.

   When a force of magnitude 8·4 N, acting parallel to the plane, is applied to the packet as shown, the packet is just on the point of moving up the plane. Modelling the packet as a particle,

   (a) show that $7(\cos\alpha + 8\sin\alpha) = 40$. **(6 marks)**

   Given that the solution of this equation is $\alpha = 38°$,

   (b) find the acceleration with which the packet moves down the plane when it is released from rest with no external force applied. **(4 marks)**

4. A car moves in a straight line from P to Q, a distance of 420 m, with constant acceleration. At P the speed of the car is 8 ms^{-1}. At Q the speed of the car is 20 ms^{-1}. Find

   (a) the time taken to travel from P to Q, **(2 marks)**

   (b) the acceleration of the car, **(2 marks)**

   (c) the time taken for the car to travel 240 m from P. **(4 marks)**

   Given that the mass of the car is 1200 kg and the tractive force of the car is 900 N,

   (d) find the magnitude of the resistance to the car's motion. **(3 marks)**

5. Two smooth spheres X and Y, of masses x kg and y kg respectively, are free to move in a smooth straight groove in a horizontal table. X is projected with speed 6 ms^{-1} towards Y, which is stationary. After the collision X moves with speed 2 ms^{-1} and Y moves with speed 3 ms^{-1}.

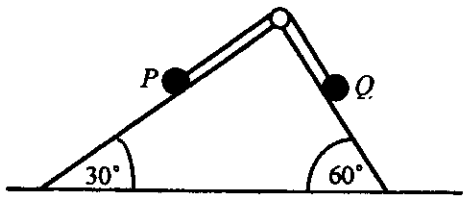
(a) Calculate the two possible values of the ratio $x : y$. **(6 marks)**

(b) State a modelling assumption that you have made concerning X and Y. **(1 mark)**

Y now strikes a vertical barrier and rebounds along the groove with speed k ms^{-1}, colliding again with X which is still moving at 2 ms^{-1}. Given that in this impact Y is brought to rest and the direction of motion of X is reversed,

(c) show that $k > 1{\cdot}5$. **(4 marks)**

6. Two particles P and Q, of masses 3 kg and 2 kg respectively, rest on the smooth faces of a wedge whose cross-section is a triangle with angles 30°, 60° and 90°, as shown. P and Q are connected by a light string, parallel to the lines of greatest slope of the two planes, which passes over a fixed pulley at the highest point of the wedge.



The system is released from rest with P 0·8 m from the pulley and Q 1 m from the bottom of the wedge, and Q starts to move down. Calculate

(a) the acceleration of either particle, **(5 marks)**

(b) the tension in the string, **(2 marks)**

(c) the speed with which P reaches the pulley. **(3 marks)**

Two modelling assumptions have been made about the string and the pulley.

(d) State these two assumptions and briefly describe how you have used each one in your solution. **(4 marks)**

7. Two stones are projected simultaneously from a point O on horizontal ground. Stone A is thrown vertically upwards with speed 98 ms^{-1}. Stone B is projected along the smooth ground in a straight line at 24·5 ms^{-1}.

(a) Find the distances of the two stones from O after t seconds, where $0 \leq t \leq 20$. **(3 marks)**

(b) Show that the distance d m between the two stones after t seconds is given by

$$d^2 = 24{\cdot}01\,(t^4 - 40t^3 + 425t^2).$$ **(6 marks)**

(c) Hence find the range of values of t for which the distance between the stones is decreasing. **(6 marks)**

