GCE Examinations

Advanced Subsidiary / Advanced Level

Pure Mathematics

Module P6

Paper E

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

Written by Rosemary Smith & Shaun Armstrong

P6 Paper E – Marking Guide

1.	*(a)*	$\lvert z-(-1+\text{i})\rvert = 2$ or $\lvert z+1-\text{i}\rvert = 2$	M1 A1	
	(b)	$y = 0 \;\therefore\; \lvert x+1-\text{i}\rvert = 2$	M1	
		$(x+1)^2 + 1 = 4$	A1	
		giving $x = -1 \pm \sqrt{3}$	A1	**(5)**

2.	assume true for $n = k \;\therefore\; 2^k > 2k$		
	$\therefore\; 2^{k+1} > 2 \times 2k$	M1 A1	
	$\therefore\; 2^{k+1} > 2k + 2k$		
	$\therefore\; 2^{k+1} > 2k + 2$ as $2k > 2$ for $k \geq 3$	M1	
	$\therefore\; 2^{k+1} > 2(k+1)$	A1	
	$\therefore$ true for $n = k+1$ if true for $n = k$		
	if $n = 3$, $2^3 = 8$, $2 \times 3 = 6 \;\therefore\; 2^3 > 2 \times 3$ so true for $n = 3$	B1	
	$\therefore$ by induction true for integers n, $n \geq 3$	A1	**(6)**

3.	*(a)*	$\ln(1+2x) - 2x\text{e}^{-x} = 2x - \frac{1}{2}(2x)^2 + \frac{1}{3}(2x)^3 + \ldots - 2x(1 - x + \frac{1}{2!}x^2 + \ldots)$	M1 A1	
		$= 2x - 2x^2 + \frac{8}{3}x^3 - 2x + 2x^2 - x^3 + \ldots$	M1	
		$\therefore\; \ln(1+2x) \approx \frac{5}{3}x^3$ so $A = \frac{5}{3}$	A1	
	(b)	$\lim\limits_{x\to 0}\left(\frac{\ln(1+2x)-2x\text{e}^{-x}}{x^3}\right) = \lim\limits_{x\to 0}\left(\frac{\frac{5}{3}x^3+kx^4+\ldots}{x^3}\right) = \frac{5}{3}$	M1 A1	**(6)**

4.	*(a)*	$\begin{pmatrix} 2 & -1 & 1 \\ 0 & 1 & -1 \\ -3 & 3 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 1 \times \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \therefore \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ is eigenvector, eigenvalue = 1	M1 A1	
	(b)	$\begin{vmatrix} 2-\lambda & -1 & 1 \\ 0 & 1-\lambda & -1 \\ -3 & 3 & 1-\lambda \end{vmatrix} = 0$	M1	
		$(2-\lambda)[(1-\lambda)^2 + 3] + 1(0-3) + 1[0 + 3(1-\lambda)] = 0$	A1	
		$(2-\lambda)(1-\lambda)^2 + 6 - 3\lambda - 3 + 3 - 3\lambda = 0$	M1	
		$(2-\lambda)(1-\lambda)^2 + 6(1-\lambda) = 0$		
		$(1-\lambda)[(2-\lambda)(1-\lambda) + 6] = 0$	A1	
		$\therefore\; \lambda = 1$ or $\lambda^2 - 3\lambda + 8 = 0$	M1	
		"$b^2 - 4ac$" $= 9 - 32 = -23$ $(< 0) \Rightarrow$ no real roots		
		$\therefore$ only 1 real eigenvalue	A1	**(8)**

5.	*(a)*	$\text{Im}(z) = 2 \;\therefore\; y = 2$		
		$u + \text{i}v = (x + 2\text{i})^2 = x^2 + 4x\text{i} - 4$	M1 A1	
		$u = x^2 - 4,\; v = 4x$	M1	
		$x = \frac{v}{4}$		
		$\therefore\; u = \frac{v^2}{16} - 4$ or $v^2 = 16(u + 4)$ which is a parabola	M1 A1	
	(b)	$\arg w = \arg(z^2) = 2\arg z \;\therefore\; \arg w = \frac{\pi}{2}$	M1 A1	
	(c)	$\arg w = \frac{\pi}{2} \Rightarrow u = 0,\, v \geq 0$	M1	
		$\therefore\; v^2 = 16(0 + 4) = 64,\; v \geq 0 \;\therefore\; v = 8$	M1 A1	
		P represents $0 + 8\text{i}$	A1	**(11)**

6.	*(a)*	(i) $\frac{\text{d}^2y}{\text{d}x^2} = 2x + \frac{\text{d}y}{\text{d}x}\cos x - y\sin x$	M1 A1	
		$\frac{\text{d}^3y}{\text{d}x^3} = 2 + \frac{\text{d}^2y}{\text{d}x^2}\cos x - \frac{\text{d}y}{\text{d}x}\sin x - \frac{\text{d}y}{\text{d}x}\sin x - y\cos x$	M1 A2	
		(ii) $y_0 = 1,\; \left(\frac{\text{d}y}{\text{d}x}\right)_0 = 1,\; \left(\frac{\text{d}^2y}{\text{d}x^2}\right)_0 = 1,\; \left(\frac{\text{d}^3y}{\text{d}x^3}\right)_0 = 2$	A1	
		$\therefore\; y = 1 + x + \frac{1}{2!}x^2 + 2(\frac{1}{3!})x^3 + \ldots = 1 + x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \ldots$	M1 A1	
		(iii) $x = -0.1,\; y \approx 0.904666\ldots = 0.905$ (3sf)	M1 A1	
	(b)	$\frac{y_1 - y_{-1}}{0.2} = x_0{}^2 + y_0\cos x_0$	M1	
		$y_1 = 0.2(x_0{}^2 + y_0\cos x_0) + y_{-1}$	A1	
		$x_{-1} = -0.1,\, x_0 = 0,\, x_1 = 0.1;\; y_{-1} = 0.904666\ldots,\, y_0 = 1,\, y_1 = ?$		
		$y_1 = 0.2(0 + 1) + 0.904666\ldots = 1.104666\ldots = 1.10$ (3sf)	A1	**(13)**

7.	*(a)*	$\overrightarrow{AB} = 2\mathbf{i} + \mathbf{j} + 5\mathbf{k},\ \overrightarrow{AC} = -\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$	M1 A1	
		$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 5 \\ -1 & -2 & -4 \end{vmatrix}$		
		$= \mathbf{i}(-4 + 10) - \mathbf{j}(-8 + 5) + \mathbf{k}(-4 + 1) = 6\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$	M1 A1	
		$\mathbf{n} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$		
		eqn. of plane is $\mathbf{r}.(2\mathbf{i} + \mathbf{j} - \mathbf{k}) = (\mathbf{i} + \mathbf{j}).(2\mathbf{i} + \mathbf{j} - \mathbf{k}) = 2 + 1 + 0 = 3$	M1 A1	
		$\mathbf{r}.(2\mathbf{i} + \mathbf{j} - \mathbf{k}) = 3$		
	(b)	$\mathbf{r} = -2\mathbf{i} - 5\mathbf{j} + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$	A1	
	(c)	at E $[(-2 + 2\lambda)\mathbf{i} + (-5 + \lambda)\mathbf{j} - \lambda\mathbf{k}].(2\mathbf{i} + \mathbf{j} - \mathbf{k}) = 3$	M1	
		$-4 + 4\lambda - 5 + \lambda + \lambda = 3$ giving $\lambda = 2$	M1 A1	
		$\therefore E$ is $(2, -3, -2)$	A1	
	(d)	E is midpoint of DF; $D(-2, -5, 0)$ $E(2, -3, -2)$ $\therefore F(6, -1, -4)$	M1 A1	**(13)**
8.	*(a)*	$\det \mathbf{M} = 2(0 - 2) - 1(0 - 2) - 1(0 - 6) = -4 + 2 + 6 = 4$	M1 A1	
		matrix of cofactors: $\begin{pmatrix} -2 & 2 & -6 \\ -2 & 2 & -2 \\ 4 & -2 & 6 \end{pmatrix}$	M1 A1	
		$\therefore \mathbf{M}^{-1} = \frac{1}{4}\begin{pmatrix} -2 & -2 & 4 \\ 2 & 2 & -2 \\ -6 & -2 & 6 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -1 & -1 & 2 \\ 1 & 1 & -1 \\ -3 & -1 & 3 \end{pmatrix}$	M1 A1	
	(b)	$\frac{x-1}{3} = \frac{y+1}{-3} = \frac{z}{4} = t$		
		$\therefore x = 3t + 1,\ y = -3t - 1,\ z = 4t$	M1 A1	
		$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -1 & -1 & 2 \\ 1 & 1 & -1 \\ -3 & -1 & 3 \end{pmatrix}\begin{pmatrix} 3t+1 \\ -3t-1 \\ 4t \end{pmatrix}$		
		$= \frac{1}{2}\begin{pmatrix} -3t-1+3t+1+8t \\ 3t+1-3t-1-4t \\ -9t-3+3t+1+12t \end{pmatrix} = \begin{pmatrix} 4t \\ -2t \\ 3t-1 \end{pmatrix}$	M1 A2	
		$\therefore x = 4t,\ y = -2t,\ z = 3t - 1$		
		giving $(t =)\ \frac{x}{4} = \frac{y}{-2} = \frac{z+1}{3}$	M1 A1	**(13)**

Total **(75)**

Performance Record – P6 Paper E

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	Total
Topic(s)	complex loci	proof by induction	Maclaurin series	matrices, eigenvals.	complex trans.	diff. eqn., Taylor series, step-by-step	vectors	matrices, inverse, trans.	
Marks	5	6	6	8	11	13	13	13	75
Student									