GCE Examinations

Pure Mathematics
Module P6

Advanced Subsidiary / Advanced Level

Paper C

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Rosemary Smith & Shaun Armstrong

1. Given that y satisfies the differential equation

$$\frac{dy}{dx} = e^x \cosh(2y + x), \text{ with } y = 1 \text{ at } x = 1,$$

(a) use the approximation $\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_0}{h}$ to obtain an estimate for y at $x = 1.01$,

(3 marks)

(b) use the approximation $\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_{-1}}{2h}$ to obtain an estimate for y at $x = 0.99$

(3 marks)

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2. The points A, B and C have coordinates $(2, 1, -1)$, $(-2, 4, -2)$ and $(a, -5, 1)$ respectively, relative to the origin O, where $a \neq 10$.

(a) Find $\overrightarrow{AB} \times \overrightarrow{AC}$. **(4 marks)**

The area of triange ABC is $4\sqrt{10}$ square units.

(b) Find the possible values of the constant a. **(3 marks)**

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3. *(a)* Given that $z = \cos\theta + i\sin\theta$, show that

$$z^n + \frac{1}{z^n} = 2\cos n\theta$$

where n is a positive integer. **(2 marks)**

The equation $5z^4 - 11z^3 + 16z^2 - 11z + 5 = 0$ has no real roots.

(b) Use the result in part *(a)* to solve the equation, giving your answers in the form $a + ib$ where $a, b \in \mathbb{R}$.

(8 marks)

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4. Given that $\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$,

(a) prove by induction that

$$\mathbf{A}^n = \begin{pmatrix} 1 & n & \tfrac{1}{2}n(n+1) \\ 0 & 1 & n \\ 0 & 0 & 1 \end{pmatrix}$$

for all positive integers n. **(6 marks)**

(b) Find the inverse of $\mathbf{A}^n$. **(5 marks)**

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5. Given that

$$\text{f}(x) = \arccos x, \quad -1 \leq x \leq 1,$$

show that

(a) $\text{f}'(x) = \dfrac{-1}{(1-x^2)^{\frac{1}{2}}}$, **(3 marks)**

(b) $(1 - x^2)\text{f}''(x) - x\text{f}'(x) = 0$. **(3 marks)**

(c) Use Maclaurin's theorem to find the expansion of f(x) in ascending powers of x up to and including the term in x^3.

(5 marks)

Turn over

6. The eigenvalues of the matrix

$$\mathbf{M} = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

are λ_1, λ_2 and λ_3.

(a) Show that $\lambda_1 = 2$ is an eigenvalue of **M** and find the other two eigenvalues λ_2 and λ_3.

(7 marks)

(b) Find an eigenvector corresponding to the eigenvalue 2. **(4 marks)**

Given that $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ are eigenvectors of **M** corresponding to λ_2 and λ_3 respectively,

(c) write down a matrix **P** such that $\mathbf{P}^{-1}\mathbf{MP} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$. **(3 marks)**

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7. The complex number $z = x + \mathrm{i}y$, where x and y are real, satisfies the equation

$$|\, z + 1 + 8\mathrm{i} \,| = 3\,|\, z + 1 \,|.$$

The complex number z is represented by the point P in the Argand diagram.

(a) Show that the locus of P is a circle and state the centre and radius of this circle.

(7 marks)

(b) Represent on the same Argand diagram the loci

$$|\, z + 1 + 8\mathrm{i} \,| = 3\,|\, z + 1 \,| \quad \text{and} \quad |\, z \,| = |\, z - \tfrac{14}{5} \,|$$ **(4 marks)**

(c) Find the complex numbers corresponding to the points of intersection of these loci, giving your answers in the form $a + \mathrm{i}b$ where a and b are real.

(5 marks)

END