GCE Examinations

Pure Mathematics Module P6

Advanced Subsidiary / Advanced Level

Paper B

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 8 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Rosemary Smith & Shaun Armstrong

1. Given that x is so small that terms in x^3 and higher powers of x may be neglected, find the values of the constants a and b for which

$$\frac{\ln(1+ax)}{1+bx} = 3x + \tfrac{3}{2}x^2.$$ **(5 marks)**

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2. Given that

$$|z + 1 - 4\text{i}| = 1,$$

(a) sketch, in an Argand diagram, the locus of z, **(2 marks)**

(b) find the maximum value of arg z in degrees to one decimal place. **(3 marks)**

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3. *(a)* Show that

$$\cosh \text{i}x = \cos x \qquad \text{where } x \in \mathbb{R}.$$ **(2 marks)**

(b) Hence, or otherwise, solve the equation

$$\cosh \text{i}x = \text{e}^{\text{i}x}$$

for $0 \le x < 2\pi$. **(3 marks)**

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4. Given that

$$u_{n+2} = 5u_{n+1} - 6u_n \quad \text{for } n \ge 1, \quad u_1 = 2 \text{ and } u_2 = 4,$$

prove by induction that $u_n = 2^n$ for all integers n, $n \ge 1$. **(6 marks)**

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5. $$\mathbf{M} = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & -4 \\ x & 3 & -1 \end{pmatrix}.$$

(a) Given that $\lambda = -1$ is an eigenvalue of **M**, find the value of x. **(3 marks)**

(b) Show that $\lambda = -1$ is the only real eigenvalue of **M**. **(6 marks)**

(c) Find an eigenvector corresponding to the eigenvalue $\lambda = -1$. **(2 marks)**

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6. A student is looking at different methods of solving the differential equation

$$\frac{dy}{dx} = xy \quad \text{with} \quad y = 1 \text{ at } x = 0.2$$

The first method the student tries is to use the approximation $\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_0}{h}$ twice with a step length of 0.1 to obtain an estimate for y at $x = 0.4$

(a) Find the value of the student's estimate for y at $x = 0.4$ **(6 marks)**

The student then realises that the exact value of y at $x = 0.4$ can be found using integration.

(b) Use integration to find the exact value of y at $x = 0.4$ **(4 marks)**

(c) Find, correct to 1 decimal place, the percentage error in the estimated value in part *(a)*.

(2 marks)

Turn over

7. *(a)* Given that $z = \cos\theta + \mathrm{i}\sin\theta$, show that

$$z^n + \frac{1}{z^n} = 2\cos n\theta \quad \text{and} \quad z^n - \frac{1}{z^n} = 2\mathrm{i}\sin n\theta,$$

where n is a positive integer. **(3 marks)**

(b) Given that

$$\cos^4\theta + \sin^4\theta = A\cos 4\theta + B,$$

find the values of the constants A and B. **(8 marks)**

(c) Hence find the exact value of

$$\int_0^{\frac{\pi}{8}} \cos^4\theta + \sin^4\theta \;\; \mathrm{d}\theta.$$ **(3 marks)**

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8. The points A, B, C and D have coordinates $(3, -1, 2)$, $(-2, 0, -1)$, $(1, 2, 6)$ and $(-1, -5, 8)$ respectively, relative to the origin O.

(a) Find $\overrightarrow{AB} \times \overrightarrow{AC}$. **(5 marks)**

(b) Find the volume of the tetrahedron $ABCD$. **(3 marks)**

The plane Π contains the points A, B and C.

(c) Find a vector equation of Π in the form $\mathbf{r.n} = p$. **(3 marks)**

The perpendicular from D to Π meets the plane at the point E.

(d) Find the coordinates of E. **(6 marks)**

END