GCE Examinations

Advanced Subsidiary / Advanced Level

Pure Mathematics
Module P6

Paper A

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

Solomon
Press

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P6 Paper A – Marking Guide

1. *(a)* $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 1 \\ 2 & -1 & -2 \end{vmatrix}$

$= \mathbf{i}(0 + 1) - \mathbf{j}(-2 - 2) + \mathbf{k}(-1 - 0) = \mathbf{i} + 4\mathbf{j} - \mathbf{k}$ M1 A2

(b) $d = \left| \dfrac{(-4\mathbf{i}+\mathbf{j}-5\mathbf{k}).(\mathbf{i}+4\mathbf{j}-\mathbf{k})}{\sqrt{1+16+1}} \right|$ M1 A1

$= \left| \dfrac{-4+4+5}{\sqrt{18}} \right| = \dfrac{5}{\sqrt{18}}$ or $\frac{5}{6}\sqrt{2}$ A1 **(6)**

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2. assume true for $n = k$ $\therefore \sum_{r=1}^{k} (r^2 + 1)r! = k(k + 1)!$

$\therefore \sum_{r=1}^{k+1} (r^2 + 1)r! = k(k + 1)! + [(k + 1)^2 + 1](k + 1)!$ M1 A1

$= (k + 1)!(k + k^2 + 2k + 2) = (k + 1)!(k^2 + 3k + 2)$ M1

$= (k + 1)!(k + 2)(k + 1) = (k + 1)[(k + 1) + 1]!$ A1

$\therefore$ true for $n = k + 1$ if true for $n = k$

if $n = 1$ $\sum_{r=1}^{n} (r^2 + 1)r! = 2 \times 1! = 2$; $n(n + 1)! = 1 \times 2! = 2$ $\therefore$ true for $n = 1$ B1

$\therefore$ by induction true for $n \in \mathbb{Z}^+$ A1 **(6)**

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3. *(a)* $z^3 = -27$ $\therefore (re^{i\theta})^3 = 27e^{i\pi}$ M1

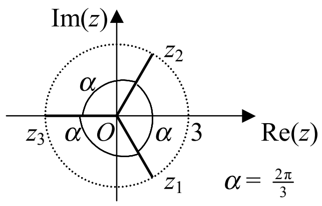
$r^3 = 27$ so $r = 3$ A1

$3\theta = 2n\pi + \pi$ M1

$n = -1, 0, 1$ gives $\theta = -\frac{\pi}{3}, \frac{\pi}{3}, \pi$ A1

$\therefore z_1 = 3e^{-i\frac{\pi}{3}}, z_2 = 3e^{i\frac{\pi}{3}}, z_3 = 3e^{i\pi}$ A1

(b)



$\alpha = \frac{2\pi}{3}$ B2 **(7)**

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4. *(a)* $\begin{vmatrix} 2-\lambda & a \\ 2 & b-\lambda \end{vmatrix} = 0$ $\therefore (2 - \lambda)(b - \lambda) - 2a = 0$ M1 A1

$\lambda_1 = -2$ gives $4b - 2a + 8 = 0$

$\lambda_2 = 3$ gives $-b - 2a + 3 = 0$ M1

solve simul. giving $a = 2, b = -1$ A1

(b) $\lambda_1 = -2$, $\begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\begin{matrix} 4x+2y=0 \\ \therefore y = -2x \end{matrix}$ $\therefore$ eigenvector $k\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ M1 A1

$\lambda_1 = 3$, $\begin{pmatrix} -1 & 2 \\ 2 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\begin{matrix} -x+2y=0 \\ \therefore x = 2y \end{matrix}$ $\therefore$ eigenvector $k\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ A1

(c) $\mathbf{P} = \dfrac{1}{\sqrt{5}}\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$ M1 A1 **(9)**

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5. $x_0 = -1,\ y_0 = 1,\ \left(\frac{dy}{dx}\right)_0 = 1;\ 2\left(\frac{d^2y}{dx^2}\right)_0 - 4 + 2 = 0\ \therefore\ \left(\frac{d^2y}{dx^2}\right)_0 = 1$ M1 A1

$(1 + x^2)\frac{d^3y}{dx^3} + 2x\frac{d^2y}{dx^2} + 4x\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 2\frac{dy}{dx} = 0$ M1 A1

$(1 + x^2)\frac{d^3y}{dx^3} + 6x\frac{d^2y}{dx^2} + 6\frac{dy}{dx} = 0$ A1

$2\left(\frac{d^3y}{dx^3}\right)_0 - 6 + 6 = 0\ \therefore\ \left(\frac{d^3y}{dx^3}\right)_0 = 0$ A1

$(1 + x^2)\frac{d^4y}{dx^4} + 2x\frac{d^3y}{dx^3} + 6x\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 6\frac{d^2y}{dx^2} = 0$ M1 A1

$2\left(\frac{d^4y}{dx^4}\right)_0 + 0 + 12 = 0\ \therefore\ \left(\frac{d^4y}{dx^4}\right)_0 = -6$ A1

$y = 1 + 1(x + 1) + \frac{1}{2!}(x + 1)^2 - \frac{6}{4!}(x + 1)^4 + ...$ M1

$y = 1 + (x + 1) + \frac{1}{2}(x + 1)^2 - \frac{1}{4}(x + 1)^4 + ...$ A1 **(11)**

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6. $\frac{y_1 - 2y_0 + y_{-1}}{0.01} = x_0 \times \frac{y_1 - y_{-1}}{0.2} + y_0^2$ M1 A1

$20y_1 - 40y_0 + 20y_{-1} = x_0 y_1 - x_0 y_{-1} + 0.2y_0^2$ M1

$y_1(20 - x_0) = 40y_0 - 20y_{-1} - x_0 y_{-1} + 0.2y_0^2$ M1

$y_1 = \frac{40y_0 - 20y_{-1} - x_0 y_{-1} + 0.2y_0^2}{20 - x_0}$ A1

$x_{-1} = 0.1,\ x_0 = 0.2,\ x_1 = 0.3;\ \ y_{-1} = 1.2,\ y_0 = 0.9,\ y_1 = ?$

$y_1 = \frac{36 - 24 - 0.24 + 0.162}{19.8} = 0.60212... = 0.602$ (3sf) M1 A2

$y_2 = \frac{40y_1 - 20y_0 - x_1 y_0 + 0.2y_1^2}{20 - x_1}$ M1

$x_0 = 0.2,\ x_1 = 0.3,\ x_2 = 0.4;\ \ y_0 = 0.9,\ y_1 = 0.60212\ldots,\ y_2 = ?$

$y_2 = \frac{24.08... - 18 - 0.27 + 0.0725...}{19.7} = 0.29885... = 0.299$ (3sf) M1 A1 **(11)**

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7. *(a)* det $\mathbf{M} = 2(8 - 3k) - 1(2k + 3) + 1(k^2 + 4) = k^2 - 8k + 17$ M1 A1

(b) det $\mathbf{M} = (k - 4)^2 - 16 + 17 = (k - 4)^2 + 1$ M1

$(k - 4)^2 \geq 0 \;\therefore$ det $\mathbf{M} > 0 \;\therefore\; \mathbf{M}$ non-singular for all real k A1

(c) $k = 3,\; \mathbf{M} = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 4 & 3 \\ -1 & 3 & 2 \end{pmatrix}$, det $\mathbf{M} = 2$ B1

matrix of cofactors: $\begin{pmatrix} -1 & -9 & 13 \\ 1 & 5 & -7 \\ -1 & -3 & 5 \end{pmatrix}$ M1 A1

$\therefore\; \mathbf{M}^{-1} = \frac{1}{2}\begin{pmatrix} -1 & 1 & -1 \\ -9 & 5 & -3 \\ 13 & -7 & 5 \end{pmatrix}$ A1

(d) $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \mathbf{M}^{-1}\begin{pmatrix} 0 \\ 3 \\ 5 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -1 & 1 & -1 \\ -9 & 5 & -3 \\ 13 & -7 & 5 \end{pmatrix}\begin{pmatrix} 0 \\ 3 \\ 5 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$

$a = -1,\; b = 0,\; c = 2$ M1 A2 **(11)**

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8. *(a)* $w(\text{i}z - 1) = z + 1;\;\; \text{i}wz - w = z + 1$ M1

$z(\text{i}w - 1) = w + 1 \;\therefore\; z = \frac{w+1}{\text{i}w-1}$ A1

$|\,z\,| = 1 \;\therefore\; |\,w + 1\,| = |\,\text{i}w - 1\,|$ M1

$|\,w + 1\,| = |\,\text{i}(w + \text{i})\,| = |\,\text{i}\,|\,|\,w + \text{i}\,| = |\,w + \text{i}\,|$

$\therefore$ perp. bisector of -1 and $-\text{i}$ $\;\therefore\; u = v$ M1 A1

(b) $\text{Im}\, z = 0 \;\therefore\; y = 0$ so $u + \text{i}v = \frac{x+1}{\text{i}x-1}$ M1

$(u + \text{i}v)(-1 + \text{i}x) = x + 1$

$-u - vx + \text{i}(ux - v) = x + 1$ M1

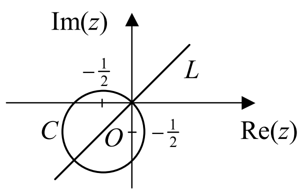
$-u - vx = x + 1 \;\therefore\; x = \frac{-u-1}{1+v};\;\; ux - v = 0 \;\therefore\; x = \frac{v}{u}$ A1

$\therefore\; \frac{-u-1}{1+v} = \frac{v}{u};\;\; -u^2 - u = v + v^2$ M1

giving $(u + \frac{1}{2})^2 + (v + \frac{1}{2})^2 = \frac{1}{2}$ A1

$\therefore$ circle, centre $-\frac{1}{2} - \frac{1}{2}\text{i}$, radius $\frac{1}{\sqrt{2}}$ A1

(c)



B3 **(14)**

Total **(75)**

Performance Record – P6 Paper A

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	Total
Topic(s)	vectors	proof by induction	complex nos.	matrices, eigenvals.	Taylor series soln. of diff. eqn.	step-by-step soln. of diff. eqn.	matrices, inverse	complex trans.	
Marks	6	6	7	9	11	11	11	14	75
Student									