1. The points A and B in the Argand diagram represent the complex numbers i and $2 - \mathrm{i}$ respectively.
   Write down an equation, in complex number form, to describe the locus of a point P whose distance from A is twice its distance from B. **(3 marks)**

2. Use the step-by-step formula $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 \approx \frac{y_1 - y_{-1}}{2h}$ to estimate the value of y when $x = 0{\cdot}2$, given that $\frac{\mathrm{d}y}{\mathrm{d}x} = x + 2y$, $y = 0$ when $x = 0$, and $y = 0{\cdot}005$ when $x = 0{\cdot}1$. **(5 marks)**

3. Use the method of induction to prove that, for any positive integer n, $7^{2n} - 5$ is a multiple of 4. **(7 marks)**

4. (a) Find the first three non-zero terms in the Maclaurin series expansion of $\ln(1 + x + x^2)$, where $|x + x^2| < 1$. **(5 marks)**
   (b) Use your answer to write down the first three non-zero terms in the expansion of
   $$\ln\left(\frac{1}{1+x+x^2}\right).$$ **(3 marks)**

5. The plane Π has equation $\mathbf{r}.(\mathbf{i} + \mathbf{j} - 4\mathbf{k}) = 8$.
   A is the point (4, 3, 2) and B is the image of A under reflection in Π.
   (a) Write down a vector in the direction of $\overrightarrow{AB}$, and hence find an equation of the line AB in the form $\mathbf{r} = \mathbf{u} + t\mathbf{v}$. **(3 marks)**
   (b) Find the co-ordinates of N, the foot of the perpendicular from A onto Π. **(5 marks)**
   (c) Hence find the co-ordinates of B. **(3 marks)**

6. (a) Find a series solution of the differential equation
   $$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - y\frac{\mathrm{d}y}{\mathrm{d}x} + 3\mathrm{e}^{-x} = 0; \text{ when } x = 0, y = 2 \text{ and } \frac{\mathrm{d}y}{\mathrm{d}x} = 1,$$
   in ascending powers of x up to and including the term in x^3. **(6 marks)**
   (b) Given instead that when $x = -1$, $y = 3$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 2$, find a series solution in ascending powers of $(x + 1)$ up to and including the term in $(x + 1)^2$. **(5 marks)**

7. (a) Find the eigenvalues, and a corresponding eigenvector in each case, of the matrix

$$\mathbf{M} = \begin{pmatrix} -1 & 1 & 1 \\ 2 & 3 & 2 \\ 0 & 1 & 0 \end{pmatrix}.$$ **(11 marks)**

The matrix **M** represents the linear transformation T of $\mathbb{R}^3$.

(b) Find cartesian equations of the invariant lines of T which pass through the origin.

(2 marks)

8. (a) Use the method of mathematical induction to prove de Moivre's theorem. **(6 marks)**

(b) Use de Moivre's theorem to show that $16 \sin^5 x = \sin 5x - 5 \sin 3x + 10 \sin x$.

(7 marks)

(c) Hence evaluate $\int_0^{\pi} \sin^5 x \, dx$. **(4 marks)**