1. Sketch the locus in the Argand diagram given by the equation $\arg(z - 2i) = \frac{\pi}{3}$. **(3 marks)**

2. Use the approximation formula $\left(\frac{d^2y}{dx^2}\right)_0 \approx \frac{y_1 - 2y_0 + y_{-1}}{h^2}$ with $h = 0{\cdot}5$ to estimate the value of y when $x = 0$, given that

$$\frac{d^2y}{dx^2} + 2xy = 3x,$$

$y = -1$ when $x = -1$, and $y = 0$ when $x = -0{\cdot}5$. **(5 marks)**

3. Prove by induction that, for all positive integers n, $n^3 - n$ is a multiple of 6. **(6 marks)**

4. (a) Find the first four terms in the expansion of $\ln\left(\frac{2+x}{1-x}\right)$ in ascending powers of x. **(5 marks)**

   (b) By putting $x = \frac{1}{2}$, deduce that $\ln\left(\frac{5}{2}\right) > \frac{57}{64}$. **(4 marks)**

5. The equation $\frac{d^2y}{dx^2} - y\frac{dy}{dx} = 0$ is satisfied by $y = f(x)$, where $f(0) = 1$ and $f'(0) = -1$.

   (a) Obtain a power series for $f(x)$ in ascending powers of x as far as the term in x^5. **(9 marks)**

   (b) Hence estimate the value of $f(0{\cdot}1)$, giving your answer to 3 decimal places. **(2 marks)**

6. (a) Show that 1 is an eigenvalue of the matrix $\mathbf{A} = \begin{pmatrix} 0 & 4 & 3 \\ -6 & 1 & 6 \\ 2 & 4 & 1 \end{pmatrix}$

   and find the other two eigenvalues of **A**. **(5 marks)**

   (b) Find an eigenvector of **A** associated with the eigenvalue 1. **(4 marks)**

   (c) Write down a diagonal matrix **D** having the property that $\mathbf{P}^{-1}\mathbf{AP} = \mathbf{D}$ for some non-singular matrix **P**, and briefly describe how you could find **P**. **(2 marks)**

7. A, B and C are the points with position vectors $\mathbf{a} = 4\mathbf{i} - \mathbf{j} - \mathbf{k}$, $\mathbf{b} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$, $\mathbf{c} = -\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ respectively. O is the origin.
   (a) Find $\mathbf{a} \times \mathbf{b}$ and $\mathbf{b} \times \mathbf{c}$. **(3 marks)**
   (b) Verify that $(\mathbf{a} \times \mathbf{b}) . \mathbf{c} = \mathbf{a} . (\mathbf{b} \times \mathbf{c})$. **(2 marks)**
   (c) Find the volume of the tetrahedron $OABC$. **(2 marks)**
   (d) Find both a cartesian and a vector equation of the plane ABC. **(7 marks)**

8. The transformation $w = 1 - \dfrac{1}{z}$ maps $z = x + iy$ to $w = u + iv$, where $z \neq 0$.
   (a) Show that if z lies on the circle $|z| = 1$ then w lies on the circle $u^2 + v^2 - 2u = 0$. **(7 marks)**
   (b) Express the equation of this circle in the form $|w - w_0| = r$, where w_0 and r are fixed real numbers. **(2 marks)**
   (c) Find the equation of the curve C whose image under the given transformation is the half-line $\arg w = \dfrac{\pi}{4}$, $w \neq 0$. Describe C fully. **(7 marks)**