PURE MATHS 6 (A) TEST PAPER 2 : ANSWERS AND MARK SCHEME

1.	$f'(x) = -\frac{1}{8-x}$ $\quad f''(x) = -\frac{1}{(8-x)^2}$ $\quad f(0) = \ln 4,\ f'(0) = -\frac{1}{8}$	B1 B1	
	$f''(0) = \frac{1}{64}$ $\quad f(x) = \ln 4 - \frac{1}{8}x - \frac{1}{128}x^2$	M1 A1	4
2.	(a) $\sinh ix = (e^{ix} - e^{-ix})/2$	B1	
	$= (\cos x + i\sin x)/2 - (\cos x - i\sin x)/2 = \sin ix$	M1 A1	
	(b) Eqn. is $i\sin x = \cos x + i\sin x$ $\quad \cos x = 0$ $\quad x = (2n+1)\pi/2$	M1 A1 M1 A1	7
3.	(a) Normals are $\mathbf{i}+\mathbf{j}+\mathbf{k},\ \mathbf{i}+2\mathbf{j}-\mathbf{k}$	B1 B1	
	(b) Line of int. has direction $(\mathbf{i}+\mathbf{j}+\mathbf{k}) \times (\mathbf{i}+2\mathbf{j}-\mathbf{k}) = -3\mathbf{i}+2\mathbf{j}+\mathbf{k}$	M1 M1 A1 A1 A1	7
4.	When $n = 1$, $2^0 = 2 - 2^0$ $\quad 1 = 2 - 1$ ✓	M1 A1	
	Assume Σ from $r = 1$ to $k = 2 - 2^{1-k}$	B1	
	Then Σ to $k+1 = 2 - 2^{1-k} + 2^{1-(k+1)} = 2 - 2^{1-k} + 2^{-k}$	M1 A1	
	$= 2 - 2^{-k}(2^1 - 1) = 2 - 2^k = 2 - 2^{1-(k+1)} = 2(1 - 2^{-(k+1)})$, q.e.d.	M1 A1 A1	8
5.	Rewrite as $z = (iw+1)/(1-w) = (iu - v + 1)/(1 - u - iv)$	M1 A1	
	$= [(1 - v + iu)(1 - u + iv)]/[(1 - u - iv)(1 - u + iv)]$	M1 A1	
	If z is real, imaginary part of this is zero	M1	
	$u(1-u) + v(1-v) = 0$ $\quad u^2 + v^2 - u - v = 0$	M1 A1 A1	8
6.	(a) $y''(0) = -1$ $\quad y''' - 2xy'' - 2y' + 2yy' = 0$	B1 M1 A1	
	$y'''(0) - 1 + 1 = 0$ $\quad y'''(0) = 0$	A1	
	(b) $y^{iv} - 2xy''' - 2y'' - 2y'' + 2yy'' + 2(y')^2 = 0$	M1 A1	
	$y^{iv}(0) + 4 - 2 + 1/2 = 0$ $\quad y^{iv}(0) = -5/2$	A1	
	$y = 1 + x/2 - x^2/2 - 5x^4/48$ $\quad y(0{\cdot}1) \approx 1{\cdot}045$ (to 4 s.f.)	M1 A1 A1	10
7.	(a) $z^n = (\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$	B1	
	$z^{-n} = (\cos\theta + i\sin\theta)^{-n} = \cos n\theta - i\sin n\theta$	B1	
	Subtracting, $z^n - z^{-n} = 2i\sin n\theta$	M1 A1	
	(b) $(z - z^{-1})^7 = z^7 + 7z^6(-z^{-1}) + 21z^5(z^{-1})^2 + 35z^4(-z^{-1})^3 + 35z^3(z^{-1})^4$ $+ 21z^2(-z^{-1})^5 + 7z(z^{-1})^6 + 35(-z^{-1})^7$	M1 A1	
	$= (z^7 - z^{-7}) - 7(z^5 - z^{-5}) + 21(z^3 - z^{-3}) - 35(z - z^{-1})$	M1 A1	
	Hence $(2i\sin\theta)^7 = 2i\sin 7\theta - 14i\sin 5\theta + 42i\sin 3\theta - 70i\sin\theta$	M1 A1	
	so $\sin^7\theta = -\frac{1}{64}(\sin 7\theta - 7\sin 5\theta + 21\sin 3\theta - 35\sin\theta)$	M1 A1	12
8.	(a) $M\begin{pmatrix} 2 & 1 & 1 \\ -1 & 3 & 4 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 4 \\ 2 & 8 & 10 \end{pmatrix}$	M1 A1	
	$M = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 4 \\ 2 & 8 & 10 \end{pmatrix} \times \begin{pmatrix} 0 & 0 & 1 \\ 4 & -1 & -9 \\ -3 & 1 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 2 & 2 & 0 \end{pmatrix}$	M1 A1 A1	
	(b) M maps $(k+1, k-1, k)$ to $(2k+1, 3k-1, 4k)$	M1 A1 A1	
	Hence eqns. of image line are: $(x-1)/2 = (y+1)/3 = z/4$	A1 A1	
	(c) Char. eqn. is $(1-\lambda)[(1-\lambda)(-\lambda) - 4] + (-2(1-\lambda)] = 0$	M1 A1	
	$(1-\lambda)[(\lambda^2 - \lambda - 6) = 0$ $\quad (1-\lambda)(\lambda-3)(\lambda+2) = 0$	M1 M1	
	$\lambda = -2, 1, 3$	A1	
	(d) E/vector for 3 is such that $x + z = 3x,\ y + 2z = 3y,\ 2x + 2y = 3z$	M1	
	Hence $z = 2x,\ y = z$ $\quad$ Eigenvector (1, 2, 2)	M1 A1	
	Normalise : unit eigenvector is $(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$	A1	19