PURE MATHS 6 (A) TEST PAPER 1 : ANSWERS AND MARK SCHEME

1.	$1 + x = 1 + x$, so true for $n = 1$	B1	
	Assume $(1 + x)^k \geq 1 + kx$ Then $(1 + x)^{k+1} = (1 + x)(1 + x)^k$	M1 A1	
	$\geq (1 + x)(1 + kx) = 1 + (k + 1)x + kx^2 > 1 + (k + 1)x$, hence result	M1 A1	5
2.	$f(0) = \sin \pi/6 = 1/2$ $f'(0) = \cos \pi/6 = (\sqrt{3})/2$ $f''(0) = -1/2$	B1 M1 A1	
	Series is $\frac{1}{2} + \frac{\sqrt{3}}{2}x - \frac{1}{4}x^2$	A1 A1	5
3.	$[16(1/2 + i\sqrt{3}/2)]^{1/4} = 2(\cos \pi/3 + i \sin \pi/4)^{1/4} = (2e^{i\pi/3})^{1/4}$	M1 M1 A1	
	$= 2e^{i\pi/12},\ 2^{7i\pi/4},\ 2e^{-5i\pi/12},\ 2e^{-11i\pi/12}$	M1 A1 A1 A1	7
4.	$\det M = 0 - x(-2) = 2x$	M1 A1 A1	
	$M^{-1} = \frac{1}{2x}\begin{pmatrix} 1 & 1 & -2 \\ -x & x & 0 \\ x & -x & 2x \end{pmatrix}$ or equivalent	M1 A1 A1 A1	7
5.	(a) $\sec x (\sec^2 x) + \tan x (\sec x \tan x) = \sec^3 x + \sec x \tan^2 x$	M1 A1	
	(b) Let $y = \sec x$ $y(0) = 1, y'(0) = 0, y''(0) = 1$	B1 B1	
	Expansion is $1 + \frac{1}{2}x^2$	M1 A1	
	(c) $\ln (\sec x + \tan x) = \int \sec x \, dx$ Integrating the series gives	B1 M1	
	$x + \frac{1}{6}x^3 + \frac{1}{24}x^5$	A1 A1	10
6.	(a) $y'(0.1) = 0{\cdot}1^2 \times 1{\cdot}2 = 0{\cdot}012$	M1 A1	
	$0{\cdot}012 \approx [y(0{\cdot}2) - 1{\cdot}2]/0{\cdot}1$ $y(0{\cdot}2) \approx 0{\cdot}0012 + 1{\cdot}2 = 1{\cdot}2012$	M1 A1 A1	
	(b) $\int dy/y = \int x^2 \, dx$ $\ln y = x^3/3 + c$ $y = A \exp(x^3/3)$	M1 A1	
	$1{\cdot}2 = A\, e^{0{\cdot}001/3}$ $A = 1{\cdot}2\, e^{-0{\cdot}001/3}$	M1 A1	
	When $x = 0{\cdot}2$, $y = 1{\cdot}2\, e^{-0{\cdot}00033}\, e^{0{\cdot}00267} = 1{\cdot}2028$	M1 A1	11
7.	(a) $\mathbf{i} - \mathbf{j} - \mathbf{k},\ 2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$	B1 B1	
	(b) $(\mathbf{i} - \mathbf{j} - \mathbf{k}) \times (2\mathbf{i} - \mathbf{j} - 2\mathbf{k}) = \mathbf{i} + \mathbf{k}$	M1 A1 A1	
	(c) $\mathbf{r} . (\mathbf{i} + \mathbf{k}) = c$ Let $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$, so $c = 5$	M1 A1	
	$\mathbf{r}.(1/\sqrt{2}\,\mathbf{i} + 1/\sqrt{2}\,\mathbf{k}) = 5/\sqrt{2}$	M1 A1	
	p is perpendicular distance from origin to plane	B1	
	(d) Find angle between normals : $(\mathbf{i} + \mathbf{k}) . (\mathbf{i} - \mathbf{j} + 2\mathbf{k}) = 3$	M1 A1	
	$\sqrt{2}\sqrt{6} \cos \theta = 3$ $\cos \theta = 3/\sqrt{12}$ $\theta = 0{\cdot}524$	M1 A1 A1	15
8.	(a) $w = (4 - i)/(2 - 2i) = (5 + 3i)/4$	M1 A1	
	(b) $z = (2w + i)/(w - 1)$	M1 A1	
	When $w = 1 - i$, $z = (2 - i)/(-i) = 1 + 2i$	M1 A1 A1	
	(c) $z = \frac{2u + (2v + 1)i}{(u-1) + vi} = \frac{(2u + (2v + 1)i)((u - 1) - vi)}{((u-1) + vi)((u - 1) - vi)}$	M1 A1	
	$= \frac{2u^2 + 2v^2 - 2u + v + i(u - 2v - 1)}{(u-1)^2 + v^2}$	M1 A1	
	If $y = -x$ then $\mathrm{Re}(z) + \mathrm{Im}(z) = 0$, so $2u^2 + 2v^2 - u - v - 1 = 0$	M1 A1 A1	
	This is a circle	A1	15