CM Question Reports

FP1 - Practice Paper A

This report

When writing my papers, I author questions for particular purposes and to help tease out key ideas and skills. This report will examine the reasoning behind the different questions of this paper and, based on the cohort of students that sat this paper, the strengths and weaknesses that were brought out.

This particular paper was sat by 37 students and the distribution of marks, along with my estimated perception of the relative difficulty of the paper[1], gave rise the following grade boundaries:

Grade	A	B	C	D	E	U
Mark	58	51	46	38	32	< 32

Question 1

Such questions are quite common on FP1 papers and so part (a) was intended to ease candidates into the paper. Most candidates correctly used the standard formulae required and made apt progress towards the given answer. Errors were seen in factorisation and candidates forgetting that the $\sum_{r=1}^{n} 1 = n$. It was good to see some very well expressed mathematics and workings that had been set out clearly. These tended to contain less mistakes and, where they did, it was much easier to give them maximum credit. Part (b) was less well done, but still completed well. Many candidates were able to recognise the link between the two parts, use the correct limits and work out the value of the desired sum. Some, however, didn't quite seem to grasp the idea and struggled with this part.

[1]The *relative difficulty* is a comparison between this paper and existing FP1 papers, an inspection of the distribution of the marks achieved in those papers and the grade boundaries that were consequently set.

Question 2

Many candidates scored full marks on this question, proving numerical methods to be a topic that they find relatively simple. The interval bisection procedure was carried out very well and neatly by many candidates, although a few seemed to suffer from careless substitutions of values into their calculators. Candidates should be reminded that they need to explicitly justify their chosen interval and show and comment on the 'change of sign' that occurs across the interval to gain maximum credit. Part (b) was also very well done. The numbers here were simpler than usual and perhaps this was the reason that many dealt with the algebra confidently. It should be noted though that a fair few candidates incurred algebraic mistakes that you would expect to see on a GCSE script - these are certainly not expected at Further Maths level. Algebra is a very important tool and candidates need to make sure they are fully confident with it.

Question 3

In part (a), B1 was scored by all the candidates and 80% of the candidates managed to score at least 3/5 marks. The use of a quartic equation gave rise to a few problems for some candidates, but the majority seemed quite confident in their workings. Algebraic division was the most recurring place for error, with some candidates adding terms or adding and taking away terms. Sign errors were also recursively seen. In part (b), the Argand diagram was often neatly drawn and 84% candidates scored full marks ft of their roots. Of the 16% that didn't score, they often confused the $\Re$ and $\Im$ axis or placed their roots in the incorrect quadrant, which was disappointing to see at this level.

Question 4

All candidates scored at least 1 mark in part (a), with only 4 candidates not scoring both. Part (b) was intended to be less familiar than usual and it was a good discriminator. The most common method was to use inverse matrices to find $\mathbf{N}$ and then compare elements to find a and b. Mistakes were often seen during the multiplication of these and some candidates didn't respect the non-commutative nature of matrix multiplication - inevitably leading to errors. Good candidates noticed that the $\frac{1}{15}$ from the inverse matrix would cancel with the determinant with $\mathbf{M}$, which made their manipulation simpler. Of those who took the alternative approach, success was much higher. This involved less steps, but errors were unfortunately seen where candidates did not carefully 'collect like terms' when multiplying $\mathbf{M}$ with $\mathbf{N}$. Overall, this question discriminated well between stronger and weaker candidates with a mean mark of 4.7.

Question 5

This question added a slightly different feel to the more typical Newton-Raphson questions, encouraging candidates to draw on GCSE and general mathematical knowledge to answer the later parts of the question. A surprising number of candidates seemed to be reluctant to use the quadratic formula in part (a), despite the indication that α and β were needed to six decimal places. Part (b) was very often successfully done with the correct algorithm being employed. Mistakes often seen in poor arithmetic, though - which was surprising, considering the ability to use a calculator in this exam. However, issues seemed to arise frequently in parts (c) and (d). Despite being exposed to the idea at GCSE and in the interval bisection and linear interpolation methods, many candidates did not know how to answer part (c); only 46% scored here. Part (d) was done more fruitfully, with many candidates knowing how to work with percentages. The first two parts had a success rate of 75%, but only 57% managed to score fulls marks across the question as a whole.

Question 6

All 37 candidates scored 2/2 in part (a), which was very pleasing to see. In part (b), a lot of good candidates did not use their answer to (a) to simplify z_1 over z_3, which was quite difficult without FP2 knowledge. However, of those who did use their answers to (a), this part seemed to be fairly typical and a source of 'easy' marks. Part (c) was less well done, with many candidates not making any clear progress. Although the idea that the $\Im = 0$ here was well understood, a lot of candidates just did not convert z_2 into the form $a + ib$. Nonetheless, candidates seem to understand and enjoy the topic of complex numbers fairly well, as this question's mode mark of 8/8 suggests.

Question 7

This unstructured conics question was found to be quite difficult by all but the best of candidates. Many candidates could not map out a fruitful approach, although many were still able to score some marks. $\frac{dy}{dx}$ was correctly calculated by 35 of the candidates and the equation of one tangent was correctly given by 29. Where it was incorrect, it was often down to algebraic slips or working out the equation of the normal. Some candidates went to unnecessary lengths to work out the equation of the second tangent, rather than using the similarity of the coordinates to their advantage. Simultaneous equations were attempted by the majority of candidates, but many were let down by poor arithmetic, cancelling or the inability to spot the difference of two squares in the y coordinate. Those who worked out x often made no further progress, expect for around 19 candidates who went on to correctly express m in terms of n. Conics question often involve complex algebraic manipulation, so I will once again stress that candidates should practice their algebra a lot more to ensure confidence. With unstructured questions like this, a set method is key and for those who worked in a logical fashion, the difficulty of the question seemed to be lowered.

Question 8

This was a tricky proof by induction question, with a fair bit of intuition needed in the inductive stage. It was much trickier than usual. Almost all candidates proved the conjecture for $n = 1$ and went on to make a suitable assumption. The inductive step was attempted by all, but at this point, many struggled to see how to prove that it was a multiple of 21. The standard approach of doing $f(k+1) - f(k)$ was difficult here and only the best of candidates handled it well. Of those who made it to that stage, the conclusive statement was often to a satisfactory standard. The mean mark for this question was 2.6.

Question 9

This simple question was designed to sandwich two tricky proof by induction questions and offer all candidates an opportunity to score. Part (a) was done correctly by 80% of the candidates and, of those 80%, 92% scored full marks in part (b). Part (b) only required one line of working and some candidates felt reluctant to give only one line for the 3 marks. Part (c) was also well done, with the mean mark for this part being 3.6. Overall, this question showcased no real concern about candidates and was done to a good standard.

Question 10

Question 10 is another tricky proof by induction question. It is not tricky in the same way question 8 is, but the extra element of factorial knowledge makes it slightly more discriminating than usual proof by induction questions. Most candidates scored the first mark for proving the statement for $i = 1$ and then attempted to use their assumptions, scoring the first three marks quite confidently. Unfortunately, many candidates reached the stage $(k+1)!(k+2)-1$ and struggled to realise that this was the same as $(k+2)!-1$. Some candidates who reached the latter still thought they hadn't completed the proof! The statement was worth two marks here, with 96% of candidates who attempted the question scoring the first (for stating true for $i = 1$). Those who showed the statement for $i = k+1$ often scored 2/2 for the statement, but unsubstantial statements that didn't convey the consensus of proof by induction did not qualify for this final mark.

Overall Comments

The difficulty of this paper seemed to be slightly higher than that of a standard FP1 exam. Many candidates found some aspects of the later questions difficult, but there still proved to be opportunity for less able candidates to demonstrate their knowledge. Complex numbers, numerical methods and series seem to be well understood by the majority, but work on algebra, conics and proof by induction still needs to be done. With proof by induction, it is not work on the idea that needs to be done, but the actual execution. It is worth, perhaps, encouraging students to use the proof by induction worksheets online and on the crashMATHS site to hone these skills, as they are quite often a large source of marks. When lost, it can have dramatic and unpleasent affects on the totals.

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