SCAN ME

crashMATHS

FP1

PRACTICE PAPER B

crashmathsworksheets

75 MARKS / 1 HOUR 30 MINUTES

1 $$f(x) = 5 - x^2 + x^3 - 2x^4$$

(a) Given that $f(x)$ has a root α in the interval $[1,3]$, use interval bisection twice to find an interval of width 0.5 containing α. **(3)**

(b) Starting with $x_0 = 1$, use the Newton-Raphson method once to find an approximation for α. Give your answer to one decimal place. **(3)**

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Question 1 continued

TOTAL 6 MARKS

2 (a) On the same axis, sketch the graphs of

(i) $C_1 : y^2 = 4ax$, $a > 0$.

(ii) $C_2 : y = \frac{c^2}{x}$

On your sketch, you should show the focus and directrix of C_1 and the asymptotes of the curve C_2.

(5)

(b) Using your sketch, state and justify the number of solutions to the equation

$$\left(\frac{c^2}{x}\right)^2 = 4ax$$

(2)

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Question 2 continued

TOTAL 7 MARKS

3 Prove by induction that, for $n \in \mathbb{Z}^+$,

$$9^n - 2^n$$

is always divisible by 7. **(5)**

Question 3 continued

TOTAL 5 MARKS

4 The matrix **M** is given by

$$\begin{pmatrix} 4 & 6 \\ -2 & 8 \end{pmatrix}$$

(a) Find M^{-1}. **(3)**

M also equal to an enlargement of scale factor 3, centre O, followed by the matrix **N**.

(b) Find **N**. **(3)**

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Question 4 continued

TOTAL 6 MARKS

5 A function f can be expressed as

$$f(x)=\left(16x^2-25\right)\left(x^2-6x+15\right)$$

(a) Find the roots to the equation $f(x)=0$. **(5)**

The product of the roots of $f(x)$ is given by the complex number z_1.

(b) Find $|z_1|$. **(4)**

(c) Find $\arg z_1$, giving your answer in degrees to two decimal places. **(2)**

Given that z_2 is a complex number such that

$$\arg\left(\frac{z_1}{z_2}\right)=-\frac{\pi}{4}$$

(d) Find z_2. **(3)**

(e) Show z_1 and z_2 on a single Argand diagram. **(2)**

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Question 5 continued

Question 5 continued

Do not write outside the box

Question 5 continued

TOTAL 16 MARKS

6 Use matrices to solve the simultaneous equations

$$2a + 5b = 3$$
$$3a + 10b = 8$$

(4)

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Question 6 continued

TOTAL 4 MARKS

7 (a) Using standard formulae, show that

$$\sum_{r=1}^{n}\left(6r^2-4r+2^r+1\right)=n^2(2n+1)+2\left(1-2^n\right)$$ **(6)**

(b) Hence, or otherwise, find an expression for

$$\sum_{2n+1}^{n^2}\left(6r^2-4r+2^r+1\right)$$ **(4)**

(c) Hence evaluate

$$\sum_{9}^{16}\left(6r^2-4r+2^r+1\right)$$ **(2)**

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Question 7 continued

Do not write outside the box

Question 7 continued

Do not write outside the box

Question 7 continued

TOTAL 12 MARKS

8 Here is a recurrence relationship.

$$x_{k+1} = 3(x_k + 1)$$

Given that $x_1 = 1$,

(a) Find x_2 and x_3. **(2)**

(b) Use the method of mathematical induction to prove that

$$x_n = \frac{5(3^{n-1}) - 3}{2}$$

for $n \in \mathbb{Z}^+$. **(6)**

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Question 8 continued

TOTAL 8 MARKS

Do not write outside the box

9 The rectangular hyperbola H has the equation

$$xy = 16$$

The three distinct points P, Q and R lie on H and have the coordinates $\left(4p, \frac{4}{p}\right)$, $\left(4q, \frac{4}{q}\right)$ and $\left(4r, \frac{4}{r}\right)$ respectively, where $p, q, r \neq 0$.

(a) Find the equation of the line PQ. **(4)**

Another point $S\left(4\sqrt{pq}, \frac{4}{\sqrt{pq}}\right)$ lies on H, where $pq > 0$.

Given that PR is parallel to PS,

(b) Show that the normal to H at the point R is perpendicular to PQ. **(7)**

Question 9 continued

Question 9 continued

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Question 9 continued

END OF PAPER **TOTAL 11 MARKS**