SCAN ME

crashMATHS

FP1

PRACTICE PAPER A

crashmathsworksheets

75 MARKS / 1 HOUR 30 MINUTES

1 (a) Use standard formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n} (3r-1)^2 = \frac{1}{2} n\left(6n^2 + 3n - 1\right)$$

for all positive integers n. **(5)**

(b) Hence, evaluate

$$2^2 + 5^2 + 8^2 + 11^2 + \ldots + 149^2$$

(2)

Question 1 continued

TOTAL 7 MARKS

2 The function $y = f(x)$ is defined such that

$$f(x) = x^3 - 4x + 2$$

Given that $f(x)$ has a root α in the interval $[-2.5, -2]$,

(a) Use interval bisection twice to obtain an interval of width 0.125 containing α. **(3)**

Given that $f(x)$ has another root β in the interval $[0, 1]$,

(b) Use linear interpolation once to estimate the value of β, giving your answer to three significant figures. **(3)**

Question 2 continued

TOTAL 6 MARKS

3 Given that $2-i$ is a root of the equation $2x^4-14x^3+51x^2-98x+85=0$,

(a) Find all of the roots of the equation. **(5)**

(b) Show these roots on a single Argand diagram. **(3)**

Question 3 continued

TOTAL 8 MARKS

4 $\mathbf{M}\begin{pmatrix}5 & -2\\ 0 & 3\end{pmatrix}$ $\mathbf{N}\begin{pmatrix}a+6 & b-a\\ a & b-2a-1\end{pmatrix}$ $\mathbf{O}\begin{pmatrix}3 & 2\\ 1 & 0\end{pmatrix}$

(a) Find $\det(\mathbf{O})$. **(2)**

Given that $\mathbf{MN} = \det(\mathbf{M}) \cdot \det(\mathbf{O}) \cdot \mathbf{O}$,

(b) Find the values of a and b. **(5)**

Question 4 continued

TOTAL 7 MARKS

5 The curve $f(x) = 5x^2 + 11x - 17$ has two roots α and β, where $\alpha > 0$.

(a) Find α and β to six decimal places. **(2)**

(b) Taking 1.25 as the first approximation to α, apply the Newton Raphson method once to $f(x)$ to obtain a second approximation to α. Give your answer to six decimal places. **(5)**

(c) State the number of decimal places to which your approximation of α is correct. **(1)**

(d) Calculate the percentage error in your approximation to α. **(2)**

Question 5 continued

Question 5 continued

Question 5 continued

TOTAL 10 MARKS

6 $z_1 = 2\left(\cos\frac{\pi}{4} + \mathrm{i}\sin\frac{\pi}{4}\right)$, $z_2 = \lambda\left(\cos\frac{\pi}{6} + \mathrm{i}\sin\frac{\pi}{6}\right)$, $z_3 = 4\left(\cos\left(-\frac{\pi}{4}\right) + \mathrm{i}\sin\left(-\frac{\pi}{4}\right)\right)$

(a) Express z_1 and z_3 in the form $a + b\mathrm{i}$, where $a, b \in \mathbf{R}$. **(2)**

(b) Simplify $\frac{z_1}{z_3}$ **(2)**

(c) Hence, or otherwise, find the value of λ for which

$$\arg\left(\frac{z_1}{z_3} - z_2\right) = \pi$$ **(4)**

Question 6 continued

TOTAL 8 MARKS

7 A parabola has equation $y^2 = 4ax$, where $a > 0$.

The tangent to the parabola at $M(am^2, 2am)$ and $N(an^4, 2an^2)$ intersect at the point R, where m is a positive constant.

Given that R lies on the line with equation $x = 3a$, find m in terms of n. **(8)**

Question 7 continued

TOTAL 8 MARKS

8 Prove by induction that, for $n \in \mathbb{Z}^+$,

$$4^{n+1} + 5^{2n-1}$$

is divisible by 21. **(5)**

Question 8 continued

TOTAL 5 MARKS

9 The rectangular hyperbola H has the equation $xy = c^2$.

(a) Show that the equation of the tangent to H at the point $P\left(ct, \frac{c}{t}\right)$ is

$$t^2y + x = 2ct$$ **(4)**

(b) Find an equation for the normal to H at the point P. **(3)**

The tangent to H at P crosses the x axis at A and the normal to H at P crosses the y axis at B.

Given that O is the origin,

(c) In terms of c and t, work out the area of the triangle OAB. **(4)**

Question 9 continued

Question 9 continued

Question 9 continued

TOTAL 11 MARKS

10 Prove by induction that, for $i \in \mathbb{Z}^+$,

$$\sum_{n=1}^{i} n \cdot n! = (i+1)! - 1$$

(6)

Question 10 continued

END OF PAPER

TOTAL 6 MARKS