Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1
Gold Level G2

Time: 1 hour 30 minutes

Materials required for examination papers	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP1), the paper reference (6667), your surname, initials and signature.

Information for Candidates

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
There are 9 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
65	56	47	39	31	23

1. $$f(x) = 2x^3 - 6x^2 - 7x - 4.$$

(*a*) Show that f(4) = 0.

(1)

(*b*) Use algebra to solve f(*x*) = 0 completely.

(4)

June 2012

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2. (i) $$\mathbf{A} = \begin{pmatrix} 2k+1 & k \\ -3 & -5 \end{pmatrix}, \text{ where } k \text{ is a constant}$$

Given that

$$\mathbf{B} = \mathbf{A} + 3\mathbf{I}$$

where **I** is the 2 × 2 identity matrix, find

(*a*) **B** in terms of *k*,

(2)

(*b*) the value of *k* for which **B** is singular.

(2)

(ii) Given that

$$\mathbf{C} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 2 & -1 & 5 \end{pmatrix}$$

and

$$\mathbf{E} = \mathbf{CD}$$

find **E**.

(2)

June 2013 (R)

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3. $$f(x) = (x^2 + 4)(x^2 + 8x + 25)$$

(*a*) Find the four roots of f(*x*) = 0.

(5)

(*b*) Find the sum of these four roots.

(2)

June 2009

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4. (*a*) Use the standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r$ to show that

$$\sum_{r=1}^{n}(r^3 + 6r - 3) = \frac{1}{4}n^2(n + 2n + 13)$$

for all positive integers *n*.

(5)

(*b*) Hence find the exact value of

$$\sum_{r=16}^{30}(r^3 + 6r - 3)\,.$$

(2)

June 2012

5.

$$\mathbf{A} = \begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix}, \text{ where } a \text{ and } b \text{ are constants.}$$

Given that the matrix **A** maps the point with coordinates (4, 6) onto the point with coordinates (2, −8),

(*a*) find the value of *a* and the value of *b*.

(4)

A quadrilateral *R* has area 30 square units.
It is transformed into another quadrilateral *S* by the matrix **A**.
Using your values of *a* and *b*,

(*b*) find the area of quadrilateral *S*.

(4)

June 2011

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6. $$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$$

The transformation represented by **B** followed by the transformation represented by **A** is equivalent to the transformation represented by **P**.

(*a*) Find the matrix **P**.

(2)

Triangle T is transformed to the triangle T' by the transformation represented by **P**.

Given that the area of triangle T' is 24 square units,

(*b*) find the area of triangle T.

(3)

Triangle T' is transformed to the original triangle T by the matrix represented by **Q**.

(*c*) Find the matrix **Q**.

(2)

June 2013 (R)

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7. $$\mathrm{f}(n) = 2^n + 6^n.$$

(*a*) Show that $\mathrm{f}(k+1) = 6\mathrm{f}(k) - 4(2^k)$.

(3)

(*b*) Hence, or otherwise, prove by induction that, for $n \in \mathbb{Z}^+$, $\mathrm{f}(n)$ is divisible by 8.

(4)

June 2010

8.

$$\mathbf{A} = \begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}$$

and **I** is the 2 × 2 identity matrix.

(*a*) Prove that

$$\mathbf{A}^2 = 7\mathbf{A} + 2\mathbf{I}$$

(2)

(*b*) Hence show that

$$\mathbf{A}^{-1} = \frac{1}{2}(\mathbf{A} - 7\mathbf{I})$$

(2)

The transformation represented by **A** maps the point P onto the point Q.

Given that Q has coordinates $(2k + 8, -2k - 5)$, where k is a constant,

(*c*) find, in terms of k, the coordinates of P.

(4)

June 2013

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9. Prove by induction, that for $n \in \mathbb{Z}^+$,

(*a*) $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^n = \begin{pmatrix} 3^n & 0 \\ 3(3^n - 1) & 1 \end{pmatrix}$

(6)

(*b*) $f(n) = 7^{2n-1} + 5$ is divisible by 12.

(6)

June 2011

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10. (i) Use the standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r$ to evaluate

$$\sum_{r=1}^{24} (r^3 - 4r)$$

(2)

(ii) Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that

$$\sum_{r=0}^{n} (r^2 - 2r + 2n + 1) = \frac{1}{6}(n+1)(n+a)(bn+c)$$

for all integers $n \geq 0$, where a, b and c are constant integers to be found.

(6)

June 2013 (R)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1. (a)	$\text{f}(x) = 2x^3 - 6x^2 - 7x - 4$	
	$\text{f}(4) = \underline{128 - 96 - 28 - 4 = 0}$	B1
		(1)
(b)	$\text{f}(4) = 0 \Rightarrow (x-4)$ is a factor.	
	$\text{f}(x) = (x-4)(2x^2 + 2x + 1)$	M1 A1
	$x = \dfrac{-2 \pm \sqrt{4 - 4(2)(1)}}{2(2)}$	
	So, $(2)\left(x^2 + x + \frac{1}{2}\right) = 0 \Rightarrow (2)\left(\left(x \pm \frac{1}{2}\right)^2 \pm k \pm \frac{1}{2}\right)\ k \neq 0 \Rightarrow x =$	M1
	$\Rightarrow x = 4,\ \dfrac{-2 \pm 2\text{i}}{4}$	A1
		(4) **[5]**
2. (i)(a)	$\mathbf{A} = \begin{pmatrix} 2k+1 & k \\ -3 & -5 \end{pmatrix},\quad \mathbf{B} = \mathbf{A} + 3\mathbf{I}$	
	$\mathbf{B} = \mathbf{A} + 3\mathbf{I} = \begin{pmatrix} 2k+1 & k \\ -3 & -5 \end{pmatrix} + 3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	M1
	$= \begin{pmatrix} 2k+4 & k \\ -3 & -2 \end{pmatrix}$	A1
		(2)
(b)	**B** is singular $\Rightarrow \det \mathbf{B} = 0$.	
	$-2(2k+4) - (-3k) = 0$	M1
	$-4k - 8 + 3k = 0$	
	$k = -8$	A1cao
		(2)
(ii)	$\mathbf{C} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 2 & -1 & 5 \end{pmatrix}, \mathbf{E} = \mathbf{CD}$	
	$\mathbf{E} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}\begin{pmatrix} 2 & -1 & 5 \end{pmatrix} = \begin{pmatrix} 4 & -2 & 10 \\ -6 & 3 & -15 \\ 8 & -4 & 20 \end{pmatrix}$	M1 A1
		(2) **[6]**

Question Number	Scheme	Marks
3. (a)	$x^2+4=0 \quad \Rightarrow \quad x=ki, \quad x=\pm 2i$	M1, A1
	Solving 3-term quadratic	M1
	$x=\frac{-8\pm\sqrt{64-100}}{2} \quad =-4+3i$ and $-4-3i$	A1 A1ft
		(5)
(b)	$2i+(-2i)+(-4+3i)+(-4-3i)=-8$	M1 A1cso
	Alternative method :	
	Expands f(x) as quartic and chooses $\pm$ coefficient of x^3	M1
	-8	A1cso
		(2) **[7]**
4. (a)	$\sum_{r=1}^{n}\left(r^3+6r-3\right)$	
	$= \underline{\frac{1}{4}n^2(n+1)^2 + 6.\frac{1}{2}n(n+1)-3n}$	M1 A1 B1
	$= \frac{1}{4}n^2(n+1)^2 + 3n^2$	
	$= \frac{1}{4}n^2\left((n+1)^2+12\right)$	dM1
	$= \frac{1}{4}n^2\left(n^2+2n+13\right)$ **(AG)**	A1 *
		(5)
(b)	$S_n = \sum_{r=16}^{30}\left(r^3+6r-3\right) = S_{30}-S_{15}$	
	$= \frac{1}{4}(30)^2\left(30^2+2(30)+13\right) - \frac{1}{4}(15)^2\left(15^2+2(15)+13\right)$	M1
	$= 203850$	A1 **cao**
		(2) **[7]**

Question Number	Scheme	Marks
5. (a)	$\mathbf{A} = \begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix}$, where a and b are constants.	
	$\mathbf{A}\begin{pmatrix} 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$	
	Therefore, $\begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix}\begin{pmatrix} 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$	M1
	So, $-16 + 6a = 2$ and $4b - 12 = -8$	M1
	Allow $\begin{pmatrix} -16+6a \\ 4b-12 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$	
	giving $a = 3$ and $b = 1$.	A1A1
		(4)
(b)	$\det \mathbf{A} = 8 - (3)(1) = 5$	M1 A1
	Area $S = (\det \mathbf{A})(\text{Area } R)$	
	Area $S = 5 \times 30 = 150$ (units)^2	M1 A1 $\sqrt{}$
		(4) **[8]**
6. (a)	$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$	
	$\mathbf{P} = \mathbf{AB} \left\{ = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} \right\}$	M1
	$\mathbf{P} = \begin{pmatrix} 1 & 4 \\ -2 & -3 \end{pmatrix}$	A1
		(2)
(b)	$\det \mathbf{P} = 1(-3) - (4)(-2)$ $\{= -3 + 8 = 5\}$	M1
	$\text{Area}(T) = \dfrac{24}{5}$ (units)^2	dM1 A1ft
		(3)
(c)	$\mathbf{QP} = \mathbf{I} \Rightarrow \mathbf{QPP^{-1}} = \mathbf{IP^{-1}} \Rightarrow \mathbf{Q} = \mathbf{P^{-1}}$	
	$\mathbf{Q} = \mathbf{P^{-1}} = \dfrac{1}{5}\begin{pmatrix} -3 & -4 \\ 2 & 1 \end{pmatrix}$	M1 A1ft
		(2) **[7]**

Question Number	Scheme		Marks
7. (a)	LHS = $f(k+1) = 2^{k+1} + 6^{k+1}$	OR RHS $= 6f(k) - 4(2^k) = 6(2^k + 6^k) - 4(2^k)$	M1
	$= 2(2^k) + 6(6^k)$	$= 2(2^k) + 6(6^k)$	A1
	$= 6(2^k + 6^k) - 4(2^k) = 6f(k) - 4(2^k)$	$= 2^{k+1} + 6^{k+1} = f(k+1)$ (*)	A1
			(3)
(b)	$n = 1$: $f(1) = 2^1 + 6^1 = 8$, **which is divisible** by 8		B1
	Either Assume $f(k)$ divisible by 8 and try to use $f(k+1) = 6f(k) - 4(2^k)$	**Or** Assume $f(k)$ divisible by 8 and try to use $f(k+1) - f(k)$ or $f(k+1) + f(k)$ including factorising $6^k = 2^k3^k$	M1
	Show $4(2^k) = 4 \times 2(2^{k-1}) = 8(2^{k-1})$ or $8\left(\frac{1}{2}2^k\right)$	$= 2^3 2^{k-3}(1 + 5.3^k)$ or	A1
	Or valid statement	$= 2^3 2^{k-3}(3 + 7.3^k)$ o.e.	
	Deduction that result is implied for $n = k + 1$ and so is true for positive integers by induction (may include $n = 1$ true here)	Deduction that result is implied for $n = k + 1$ and so is true for positive integers by induction (must include explanation of why $n = 2$ is also true here)	A1cso
			(4) **[7]**

Question Number	Scheme	Marks
8. (a)	$\mathbf{A}^2 = \begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix} = \begin{pmatrix} 44 & -14 \\ -28 & 9 \end{pmatrix}$	
	$7\mathbf{A} + 2\mathbf{I} = \begin{pmatrix} 42 & -14 \\ -28 & 7 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 44 & -14 \\ -28 & 9 \end{pmatrix}$	M1A1
	or $\mathbf{A}^2 - 7\mathbf{A} = \mathbf{A}(\mathbf{A} - 7\mathbf{I})$	
	$\mathbf{A}(\mathbf{A} - 7\mathbf{I}) = \begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} -1 & -2 \\ -4 & -6 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 2\mathbf{I}$	
		(2)
(b)	$\mathbf{A}^2 = 7\mathbf{A} + 2\mathbf{I} \Rightarrow \mathbf{A} = 7\mathbf{I} + 2\mathbf{A}^{-1}$	M1
	$\mathbf{A}^{-1} = \frac{1}{2}(\mathbf{A} - 7\mathbf{I})$ *	A1* cso
	Numerical approach award 0/2.	
		(2)
(c)	$\mathbf{A}^{-1} = \frac{1}{2}\begin{pmatrix} -1 & -2 \\ -4 & -6 \end{pmatrix}$	B1
	$\frac{1}{2}\begin{pmatrix} -1 & -2 \\ -4 & -6 \end{pmatrix}\begin{pmatrix} 2k+8 \\ -2k-5 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -2k-8+4k+10 \\ -8k-32+12k+30 \end{pmatrix}$	M1
	$\begin{pmatrix} k+1 \\ 2k-1 \end{pmatrix}$ or $(k+1,\ 2k-1)$	A1,A1
	Or: $\begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2k+8 \\ -2k-5 \end{pmatrix}$	B1
	$6x - 2y = 2k + 8$ $-4x + y = -2k - 5 \Rightarrow x = \ldots or\ y = \ldots$	M1
	$\begin{pmatrix} k+1 \\ 2k-1 \end{pmatrix}$ or $(k+1,\ 2k-1)$	A1,A1
		(4) **[8]**

Question Number	Scheme	Marks
9. (a)	$n=1$; LHS $= \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^1 = \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$	
	RHS $= \begin{pmatrix} 3^1 & 0 \\ 3(3^1-1) & 1 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$	
	As LHS = RHS, the matrix result is true for $n=1$.	B1
	Assume that the matrix equation is true for $n = k$,	
	ie. $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^k = \begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix}$	
	With $n = k+1$ the matrix equation becomes	
	$\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^k \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$	
	$= \begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$ or $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}\begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix}$	M1
	$= \begin{pmatrix} 3^{k+1}+0 & 0+0 \\ 9(3^k-1)+6 & 0+1 \end{pmatrix}$ or $\begin{pmatrix} 3^{k+1}+0 & 0+0 \\ 6.3^k+3(3^k-1) & 0+1 \end{pmatrix}$	
	$= \begin{pmatrix} 3^{k+1} & 0 \\ 9(3^k)-3 & 1 \end{pmatrix}$	A1
	$= \begin{pmatrix} 3^{k+1} & 0 \\ 3\left(3(3^k)-1\right) & 1 \end{pmatrix}$	
	$= \begin{pmatrix} 3^{k+1} & 0 \\ 3(3^{k+1}-1) & 1 \end{pmatrix}$	dM1 A1
	If the result is true for $n = k(\;\;1)$ then it is now true for $n = k+1$. (2) As the result has shown to be true for $n=1$, (3) then the result is true for all n. (4)	A1 cso
		(6)
(b)	$\text{f}(1) = 7^{2-1}+5 = 7+5 = 12$,	B1
	{which is divisible by 12}. { $\therefore$ f (n) is divisible by 12 when $n=1$. }	
	Assume that for $n=k$,	
	$\text{f}(k) = 7^{2k-1}+5$ is divisible by 12 for $k \in \square^{+}$.	
	So, $\text{f}(k+1) = 7^{2(k+1)-1}+5$	B1
	giving, $\text{f}(k+1) = 7^{2k+1}+5$	

Question Number	Scheme	Marks
	$\therefore \mathrm{f}(k+1) - \mathrm{f}(k) = \left(7^{2k+1} + 5\right) - \left(7^{2k-1} + 5\right)$	M1
	$= 7^{2k+1} - 7^{2k-1}$	
	$= 7^{2k-1}\left(7^2 - 1\right)$	M1
	$= 48\left(7^{2k-1}\right)$	A1cso
	$\therefore \mathrm{f}(k+1) = \mathrm{f}(k) + 48\left(7^{2k-1}\right)$, which is divisible by 12 as both $\mathrm{f}(k)$ and $48\left(7^{2k-1}\right)$ are both divisible by 12.(1) If the result is true for $n = k$, (2) then it is now true for $n = k+1$. (3) As the result has shown to be true for $n = 1$,(4) then the result is true for all n. (5).	A1 cso
		(6) **[12]**
10. (i)	$\sum_{r=1}^{24}\left(r^3 - 4r\right)$	
	$= \frac{1}{4}24^2(24+1)^2 - 4.\frac{1}{2}24(24+1)$	M1
	$\{= 90000 - 1200\}$	
	$= 88800$	A1 **cao** (2)
(ii)	$\sum_{r=0}^{n}\left(r^2 - 2r + 2n + 1\right)$	
	$= \underline{\frac{1}{6}n(n+1)(2n+1) - 2.\frac{1}{2}n}(n+1) + 2n(n+1) + (n+1)$	M1 A1 B1 B1
	$= \frac{1}{6}(n+1)\left\{2n^2 + n - 6n + 12n + 6\right\}$	M1
	$= \frac{1}{6}(n+1)\left\{2n^2 + 7n + 6\right\}$	
	$= \frac{1}{6}(n+1)(n+2)(2n+3)$	A1
		(6) **[8]**

Examiner reports

Question 1

This question was well done by many candidates although there were two particular places in the question where marks were lost. In part (a) virtually all successfully substituted $x = 4$ into f(x) but many failed show sufficient working to justify that f(4) = 0. Many incorrectly assumed that $3(4)^3 – 6(4)^2 – 7(4) – 4 = 0$ was enough.

In part (b) many candidates successfully established the quadratic factor using either long division, comparing coefficients or inspection and went on to solve the resulting quadratic by using the formula or completing the square. A small number made an attempt at factorising.

There were a surprising number of cases where the final mark was lost when candidates failed to give the real root as well as the complex ones or confused solving with factorising. It was quite common to see $(x-4)\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$ as a conclusion.

Question 2

A surprisingly high proportion made errors in part (a) either by multiplying by 3**I** rather than adding or adding a matrix similar to the identity, but with the 0s and 1s swapped. A majority of candidates found the determinant correctly in part (b), but the most common error seen was then equating det **B** to 1 rather than 0. This final part was the trickiest demand with many candidates failing to realise that the matrix **E** was of order 3 × 3. However, when they did get the correct dimensions, they generally calculated it accurately and scored all marks.

Question 3

A substantial minority multiplied out the two brackets which complicated the problem. Most however attempted to solve $x^2 + 4 = 0$, but there were a number of wrong answers, particularly the real answers +2 and –2. The solution of the three term quadratic was usually correct but there were errors in simplification with a substantial number of candidates losing some accuracy in part (a). Indeed the answers –4 + 6i and –4 – 6i were fairly common. In part (b) candidates were asked to find the sum of their roots. Most obtained –8, but this only gained M1A1 if it followed wrong roots. There were candidates who were unfamiliar with the term "sum" and found the product instead.

Question 4

In part (a) many candidates could start correctly by splitting the sum into three parts and substituted appropriate expressions for each sum. Those with any mistakes at this stage were unable to score any further marks. The subsequent algebra defeated some, and quite a few candidates could not establish the printed result. There were very few candidates who thought

$$\sum_{r=1}^{n} 3 = 3.$$

Part (b) was usually answered correctly by the majority of candidates but a significant number calculated $S_{30} – S_{16}$. A small minority substituted into the original cubic to find the sum.

Question 5

Many candidates were able to multiply correctly and solve the resulting equations in part (a). There were some cases where the nature of the multiplication was confused, with the (4, 6) being placed before the matrix giving the incorrect equations $-16 + 6b = 2$ and $4a - 12 = -8$.

In part (b) the property of the determinant was well known and many could correctly find the area of the quadrilateral S although some did divide by the determinant. Those unfamiliar with the property sometimes proceeded to try and transform a figure of their choice that had an area of 30 square units in an attempt to answer the question but were largely unsuccessful.

Question 6

A most common error in part (a) was to find **P** as **BA** instead of **AB,** but candidates were awarded follow through marks in the later parts of the question. The value of the determinant was usually correct, and most knew that this helped to find the area, but some candidates multiplied by their determinant in part (b) rather than dividing. In part (c) a few candidates failed to realise that $\mathbf{P}^{-1}$ was being asked for, but many were able to find the inverse correctly from their answer to part (a).

Question 7

Well explained logical explanations to both parts were rare and indicated a very good candidate.

In part (a) those who began with the RHS and attempted to reach the LHS had most success. Elegant proofs were in the minority, with many candidates forced to start with the LHS and the RHS separately, and then attempting to meet in the middle. Those who adopt this approach should be aware of the need to reach a conclusion. Considering $f(k + 1) - 6f(k)$ was a productive starting point for some, yielding neat efficient proofs.

Part (b) was probably the toughest question on the paper. Far too many candidates ignored the "hence" and just considered $f(k + 1) - f(k)$, because that is what they usually do . Of these, very few could deal convincingly with the 6^k term that resulted, and just as few returned to making $f(k + 1)$ the subject before drawing a conclusion. Of those taking the approach suggested in the question, relatively few could demonstrate convincingly that $f(k + 1)$ was divisible by 8. Many immediately reverted to the original definitions and went nowhere. One very good candidate proved that 4×2^k was divisible by 8 using induction, then used that result in the induction method to answer the question.

There was some, fortunately not too common, highly creative work with powers, but the major problems were the confused and muddled style of candidates' answers. It was rare to see A and B divisible by 8 => A ± B divisible by 8. Also an accurate, concise, end statement making reference to the result being true for all positive integers (or equivalent) was extremely rare. Yet it was clear that teachers had tried hard to instil the essentials of the argument into their pupils, with many setting out "Basis, Assumption, Induction, Conclusion" or similar. Indeed 21% of the candidates achieved full marks on this question.

Question 8

Generally the standard of responses to this question were high for the first and the last parts. Part (a) was very well done. Almost all candidates were able to square a matrix and were able to quote the identity matrix.

In part (b) however, most candidates did not realise what was expected. Many tried a numerical approach, as in part (a) and gained no marks for their effort. Of those who did attempt an algebraic solution, some wrote that $7\mathbf{A}\mathbf{A}^{-1} = 7$, and some attempted to divide by **A**. There were, however, some excellent solutions.

In part (c) there were some good attempts and only a very few candidates attempted to multiply matrices in the wrong order. The most successful solutions used the inverse matrix $\mathbf{A}^{-1}$. Many candidates chose a method involving simultaneous equations instead and were usually successful.

Question 9

Many candidates are aware of the requirements of Proof by Induction but as in previous series, success is quite variable, and there is often a lack of precision. In part (a) the B1 mark was often scored with the relatively easy task of establishing the result for $n = 1$. Most stated the assumptive step but were then unclear what to do next and simply wrote down the result with $n = k + 1$. Those who did know they had to multiply $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$ by $\begin{pmatrix} 3^k & 0 \\ 3(3^k - 1) & 1 \end{pmatrix}$ often made some progress although there was some poor use of indices. Only the better candidates could give a clear and convincing argument and then make a suitable conclusion.

Part (b) was met with less success although most could gain the first two B marks. The majority opted for the $f(k + 1) - f(k)$ approach and met with varying success. Many were not sure what to do with the resulting $7^{2k+1} - 7^{2k-1}$ expression or failed to show convincingly that it was a multiple of 12.

Question 10

Part (a) was usually correct showing that candidates were able to make correct use of the standard formulae. Part (b) proved to be demanding even for some of the best candidates. Many could not deal with the lower limit of 0 correctly and therefore failed to get an answer in the form given. Not enough candidates realised they did not have, and could not have, a factor of $(n + 1)$ from their incorrect opening statement.

Statistics for FP1 Practice Paper Gold Level G2

Qu	Max Score	Modal score	Mean %	Mean score for students achieving grade: ALL	A*	A	B	C	D	E	U
1	5		83	4.14	4.66	4.53	4.20	3.96	3.62	3.53	3.08
2	6		80	4.82	5.66	5.42	4.77	4.59	4.30	3.55	2.85
3	7		84	5.88		6.65	5.94	5.73	5.03	3.93	3.33
4	7		89	6.24	6.96	6.86	6.51	6.25	5.60	4.79	3.43
5	8		76	6.11	7.79	7.53	6.30	5.04	3.91	2.94	1.29
6	7		78	5.48	6.71	6.61	5.92	5.48	4.34	3.18	2.49
7	7		60	4.20	6.14	5.20	4.19	3.58	3.06	2.47	1.72
8	8	6	56	4.49	6.87	5.63	4.61	3.78	2.98	2.51	1.28
9	12		68	8.10	11.09	10.13	8.00	6.23	4.93	4.01	2.38
10	8		65	5.20	7.00	6.34	5.33	4.48	4.19	4.19	2.82
	75		**73**	**54.66**		**64.90**	**55.77**	**49.12**	**41.96**	**35.10**	**24.67**