Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1
Silver Level S2

Time: 1 hour 30 minutes

Materials required for examination papers	**Items included with question**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP1), the paper reference (6667), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 9 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
69	60	52	44	37	30

1. (*a*) Using the formulae for $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$, show that

$$\sum_{r=1}^{n} r(r+1)(r+3) = \frac{1}{12}n(n+1)(n+2)(3n+k),$$

where k is a constant to be found.

(7)

(*b*) Hence evaluate $\sum_{r=21}^{40} r(r+1)(r+3)$.

(2)

June 2009

2.
$$\mathrm{f}(x) = x^2 + \frac{3}{4\sqrt{x}} - 3x - 7, \quad x > 0.$$

A root α of the equation $\mathrm{f}(x) = 0$ lies in the interval [3, 5].

Taking 4 as a first approximation to α, apply the Newton-Raphson process once to $\mathrm{f}(x)$ to obtain a second approximation to α. Give your answer to 2 decimal places.

(6)

June 2012

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3. (*a*) Given that

$$\mathbf{A} = \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix},$$

(i) find $\mathbf{A}^2$,

(ii) describe fully the geometrical transformation represented by $\mathbf{A}^2$.

(4)

(*b*) Given that

$$\mathbf{B} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix},$$

describe fully the geometrical transformation represented by **B**.

(2)

(*c*) Given that

$$\mathbf{C} = \begin{pmatrix} k+1 & 12 \\ k & 9 \end{pmatrix},$$

where k is a constant, find the value of k for which the matrix **C** is singular.

(3)

June 2011

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4. $$f(x) = (4x^2 + 9)(x^2 - 2x + 5)$$

(*a*) Find the four roots of $f(x) = 0$.

(4)

(*b*) Show the four roots of $f(x) = 0$ on a single Argand diagram.

(2)

June 2013 (R)

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5. $$\mathbf{R} = \begin{pmatrix} a & 2 \\ a & b \end{pmatrix}, \text{ where } a \text{ and } b \text{ are constants and } a > 0.$$

(*a*) Find $\mathbf{R}^2$ in terms of a and b.

(3)

Given that $\mathbf{R}^2$ represents an enlargement with centre (0, 0) and scale factor 15,

(*b*) find the value of a and the value of b.

(5)

June 2009

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6. $$f(x) = \tan\left(\frac{x}{2}\right) + 3x - 6, \quad -\pi < x < \pi.$$

(*a*) Show that the equation $f(x) = 0$ has a root α in the interval [1, 2].

(2)

(*b*) Use linear interpolation once on the interval [1, 2] to find an approximation to α. Give your answer to 2 decimal places.

(3)

June 2012

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7. $\mathbf{A} = \begin{pmatrix} a & -2 \\ -1 & 4 \end{pmatrix}$, where a is a constant.

(a) Find the value of a for which the matrix $\mathbf{A}$ is singular.

(2)

$$\mathbf{B} = \begin{pmatrix} 3 & -2 \\ -1 & 4 \end{pmatrix}$$

(b) Find $\mathbf{B}^{-1}$.

(3)

The transformation represented by $\mathbf{B}$ maps the point P onto the point Q.

Given that Q has coordinates $(k - 6, 3k + 12)$, where k is a constant,

(c) show that P lies on the line with equation $y = x + 3$.

(3)

June 2009

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8. Prove by induction that, for $n \in \mathbb{Z}^+$,

(a) $\mathrm{f}(n) = 5^n + 8n + 3$ is divisible by 4,

(7)

(b) $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^n = \begin{pmatrix} 2n+1 & -2n \\ 2n & 1-2n \end{pmatrix}$.

(7)

June 2009

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9. (*a*) A sequence of numbers is defined by

$$u_1 = 8$$

$$u_{n+1} = 4u_n - 9n, n \geq 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = 4^n + 3n + 1$$

(5)

(*b*) Prove by induction that, for $m \in \mathbb{Z}^+$,

$$\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}^m = \begin{pmatrix} 2m+1 & -4m \\ m & 1-2m \end{pmatrix}$$

(5)

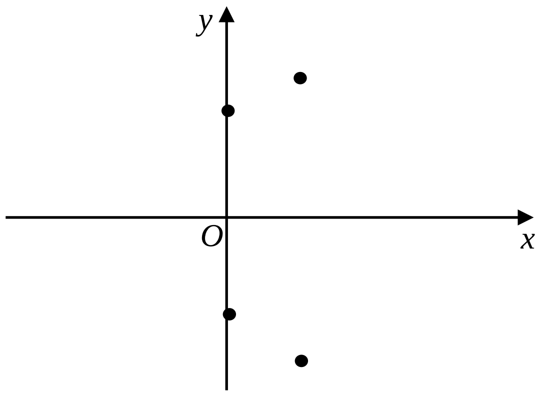
June 2013

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1. (a)	$r(r+1)(r+3) = r^3 + 4r^2 + 3r$, so **use** $\sum r^3 + 4\sum r^2 + 3\sum r$	M1
	$= \frac{1}{4}n^2(n+1)^2 + 4\left(\frac{1}{6}n(n+1)(2n+1)\right) + 3\left(\frac{1}{2}n(n+1)\right)$	A1 A1
	$= \frac{1}{12}n(n+1)\{3n(n+1) + 8(2n+1) + 18\}$ or $= \frac{1}{12}n\{3n^3 + 22n^2 + 45n + 26\}$	
	or $= \frac{1}{12}(n+1)\{3n^3 + 19n^2 + 26n\}$	M1 A1
	$= \frac{1}{12}n(n+1)\{3n^2 + 19n + 26\} = \frac{1}{12}n(n+1)(n+2)(3n+13)$ $(k = 13)$	M1 A1cao (7)
(b)	$\sum_{21}^{40} = \sum_{1}^{40} - \sum_{1}^{20}$	M1
	$= \frac{1}{12}(40 \times 41 \times 42 \times 133) - \frac{1}{12}(20 \times 21 \times 22 \times 73) = 763420 - 56210 = 707210$	A1 cao
		(2) **[9]**
2.	$f(x) = x^2 + \frac{3}{4 Ö x} - 3x - 7$, $x > 0$	
	$f(x) = x^2 + \frac{3}{4}x^{-\frac{1}{2}} - 3x - 7$	
	$f'(x) = 2x - \frac{3}{8}x^{-\frac{3}{2}} - 3 \{+ 0\}$	M1A1
	$f(4) = -2.625 = -\frac{21}{8} = -2\frac{5}{8}$ *or* $4^2 + \frac{3}{4\sqrt{4}} - 3 \times 4 - 7$	B1
	$f'(4) = 4.953125 = \frac{317}{64} = 4\frac{61}{64}$	M1
	$\alpha_2 = 4 - \left(\frac{"-2.625"}{"4.953125"}\right)$	M1
	$= 4.529968454...$ $\left(= \frac{1436}{317} = 4\frac{168}{317}\right)$	
	$= 4.53$ (2 dp)	A1 **cao** **[6]**

Question Number	Scheme	Marks
3. (a)	$\mathbf{A} = \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}$	
(i)	$\mathbf{A}^2 = \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}\begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}$	
	$= \begin{pmatrix} 1+2 & \sqrt{2}-\sqrt{2} \\ \sqrt{2}-\sqrt{2} & 2+1 \end{pmatrix}$	M1
	$= \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$	A1
(ii)	**Enlargement**; scale factor 3, centre $(0, 0)$.	B1; B1 (4)
(b)	$\mathbf{B} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	
	Reflection; in the line $y = -x$.	B1; B1 (2)
(c)	$\mathbf{C} = \begin{pmatrix} k+1 & 12 \\ k & 9 \end{pmatrix}$, k is a constant.	
	C is singular $\Rightarrow$ $\det \mathbf{C} = 0$. (Can be implied)	B1
	$9(k+1) - 12k \; (= 0)$	M1
	$9k + 9 = 12k$	
	$9 = 3k$	
	$k = 3$	A1 (3) **[9]**

Question Number	Scheme	Marks
4.	$\mathrm{f}(x) = (4x^2 + 9)(x^2 - 2x + 5) = 0$	
(a)	$(4x^2 + 9) = 0 \Rightarrow x = \frac{3i}{2}, -\frac{3i}{2}$	M1 A1
	$(x^2 - 2x + 5) = 0 \Rightarrow x = \frac{2 \pm \sqrt{4 - 4(1)(5)}}{2(1)}$	M1
	$\Rightarrow x = \frac{2 \pm \sqrt{-16}}{2}$	
	$\Rightarrow x = 1 \pm 2i$	A1
		(4)
(b)		B1ft B1ft
		(2) **[6]**
5. (a)	$\mathbf{R}^2 = \begin{pmatrix} a^2 + 2a & 2a + 2b \\ a^2 + ab & 2a + b^2 \end{pmatrix}$	M1 A1 A1
		(3)
(b)	Puts their $a^2 + 2a = 15$ or their $2a + b^2 = 15$ or their $(a^2 + 2a)(2a + b^2) - (a^2 + ab)(2a + 2b) = 225$ (or to 15) ,	M1
	Puts their $a^2 + ab = 0$ or their $2a + 2b = 0$	M1
	Solve to find either a or b	M1
	$a = 3, \quad b = -3$	A1, A1
		(5) **[8]**

Question Number	Scheme	Marks
6. (a)	$\mathrm{f}(x) = \tan\left(\frac{x}{2}\right) + 3x - 6, \quad -\pi < x < \pi$	
	$\mathrm{f}(1) = -2.45369751...$ $\mathrm{f}(2) = 1.557407725...$	M1
	Sign change (and $\mathrm{f}(x)$ is continuous) therefore a root α is between $x = 1$ and $x = 2$.	A1
		(2)
(b)	$\frac{\alpha - 1}{\text{"}2.45369751...\text{"}} = \frac{2 - \alpha}{\text{"}1.557407725...\text{"}}$ **or** $\frac{\text{"}2.45369751...\text{"} + \text{"}1.557407725\text{"}}{1} = \frac{\text{"}2.45369751...\text{"}}{\alpha - 1}$	M1
	$\alpha = 1 + \left(\frac{\text{"}2.45369751...\text{"}}{\text{"}1.557407725...\text{"} + \text{"}2.45369751...\text{"}}\right)1$	A1 $\sqrt{}$
	$= \frac{6.464802745}{4.011105235}$	
	$= 1.611726037...$	A1
		(3) **[5]**
7. (a)	Use $4a - (-2 \times -1) = 0 \quad \Rightarrow \quad a, = \frac{1}{2}$	M1, A1
		(2)
(b)	Determinant: $(3 \times 4) - (-2 \times -1) = 10 \quad (\Delta)$	M1
	$\mathbf{B}^{-1} = \frac{1}{10}\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$	M1 A1cso
		(3)
(c)	$\frac{1}{10}\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} k-6 \\ 3k+12 \end{pmatrix}, \quad = \frac{1}{10}\begin{pmatrix} 4(k-6)+2(3k+12) \\ (k-6)+3(3k+12) \end{pmatrix}$	M1, A1ft
	$\begin{pmatrix} k \\ k+3 \end{pmatrix}$ Lies on $y = x + 3$	A1
		(3) **[8]**

Question Number	Scheme	Marks
8. (a)	$\mathrm{f}(1) = 5 + 8 + 3 = 16$, (which is divisible by 4). ($\therefore$ True for n = 1).	B1
	Using the formula to write down f(k + 1), $\mathrm{f}(k+1) = 5^{k+1} + 8(k+1) + 3$	M1 A1
	$\mathrm{f}(k+1) - \mathrm{f}(k) = 5^{k+1} + 8(k+1) + 3 - 5^k - 8k - 3$	M1
	$= 5(5^k) + 8k + 8 + 3 - 5^k - 8k - 3 = 4(5^k) + 8$	A1
	$\mathrm{f}(k+1) = 4(5^k + 2) + \mathrm{f}(k)$, **which is divisible by** 4	A1ft
	$\therefore$ True for $n = k + 1$ **if** true for $n = k$. True for n = 1, $\therefore$ true for all n.	A1cso (7)
(b)	For n = 1, $\begin{pmatrix} 2n+1 & -2n \\ 2n & 1-2n \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^1$ **(**$\therefore$ True for n = 1.)	B1
	$\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 2k+1 & -2k \\ 2k & 1-2k \end{pmatrix}\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 2k+3 & -2k-2 \\ 2k+2 & -2k-1 \end{pmatrix}$	M1 A1 A1
	$= \begin{pmatrix} 2(k+1)+1 & -2(k+1) \\ 2(k+1) & 1-2(k+1) \end{pmatrix}$	M1 A1
	$\therefore$ True for $n = k + 1$ **if** true for $n = k$. **True for *n*** = 1, $\therefore$ **true for all *n***	A1 cso (7) **[14]**

Question Number	Scheme	Marks
9. (a)	$u_1 = 8$ given	
	$n = 1 \Rightarrow u_1 = 4^1 + 3(1) + 1 = 8$ $(\therefore$ true for $n = 1)$	B1
	Assume true for $n = k$ so that $u_k = 4^k + 3k + 1$	
	$u_{k+1} = 4\left(4^k + 3k + 1\right) - 9k$	M1
	$= 4^{k+1} + 12k + 4 - 9k = 4^{k+1} + 3k + 4$	A1
	$= 4^{k+1} + 3\left(k+1\right) + 1$	A1
	If <u>true for $n = k$</u> then <u>true for $n = k + 1$</u> and as <u>true for $n = 1$</u> <u>true for all n</u>	A1 cso
		(5)
(b)	**Condone use of *n* here.**	
	$lhs = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}^1 = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$	
	$rhs = \begin{pmatrix} 2(1)+1 & -4(1) \\ 1 & 1-2(1) \end{pmatrix} = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$	B1
	Assume $\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}^k = \begin{pmatrix} 2k+1 & -4k \\ k & 1-2k \end{pmatrix}$	
	$\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 2k+1 & -4k \\ k & 1-2k \end{pmatrix}\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$	M1
	$= \begin{pmatrix} 6k+3-4k & -8k-4+4k \\ 3k+1-2k & -4k-1+2k \end{pmatrix}$	A1
	$= \left(\begin{pmatrix} 2k+3 & -4k-4 \\ k+1 & -2k-1 \end{pmatrix}\right)$	
	$= \begin{pmatrix} 2\left(k+1\right)+1 & -4\left(k+1\right) \\ k+1 & 1-2\left(k+1\right) \end{pmatrix}$	A1
	If <u>true for $m = k$</u> then <u>true for $m = k + 1$</u> and as <u>true for $m = 1$</u> true <u>for all m</u>	A1 cso
		(5) **[10]**

Examiner reports

Question 1

In part (a) there were many good solutions, although some candidates increased the algebraic challenge by not extracting all common factors. There were a number of answers which went straight from a cubic to three linear factors with no evidence such as an intermediate quadratic. This question had a printed answer, which candidates were asked to show and all steps of the working are required for full marks.

The evaluation of the sum of the series in part (b) was usually done well. Most used their formula correctly and subtracted the sum of the first twenty terms from the sum of the first forty terms to give their answer. A small minority substituted into the wrong formula and a larger number misquoted their formula for the sum using a fraction ½ instead of 1/12. Very few subtracted the first 19 terms or 21 terms which has been a common error in the past.

Question 2

Many candidates gained full marks for this question. Errors usually resulted from incorrect differentiation on the second term of the expression. The most common error was to write $\frac{3}{4\sqrt{x}}$ as $3\times 4x^{-\frac{1}{2}}$ and hence obtain an incorrect derivative. Work was often clear and explicit with candidates showing both $\mathrm{f}(4)$ and $\mathrm{f}'(4)$ evaluated and substituted correctly into the Newton-Raphson formula. The minimum acceptable response required a correct derivative, a correct statement of the Newton-Raphson process and an answer to the correct accuracy. However, candidates are advised that in this type of question, full working should be shown. A small minority of candidates attempted interval bisection.

Question 3

Many candidates gained full marks for this question. In part (a)(i) most candidates were comfortable with matrix multiplication and could correctly find $\mathbf{A}^2$. In (a)(ii), it was very common to see “enlargement scale factor 3” but with no mention of the centre.

In part (b), the nature of the transformation was identified correctly although a “rotation of 180 degrees” was common. Some tried a combination of transformations with little success.

In part (c) many knew what was meant by a matrix being **singular** and proceeded to find the correct value for *k*.

Question 4

Generally both parts of this question were done very well. Errors that candidates made included $\pm\frac{3}{2}$ rather than $\pm\frac{3\mathrm{i}}{2}$ or use of an incorrect quadratic formula. Almost all the candidates were able to plot their solutions on a correct Argand diagram.

Question 5

In part (a) the product of the two matrices was usually executed correctly with few errors. Part (b) caused difficulties for some and there were a number of attempts where pages were covered in matrix work which led nowhere. The common errors included solving $\mathbf{R}^2 = 15\mathbf{R}$ instead of solving $\mathbf{R}^2 = 15\mathbf{I}$. A sizeable minority used the determinant of the matrix, putting it equal to 15 or to 225. They usually did not give a second equation to enable them to find the two unknowns. The successful majority approached the solution by equating the elements of their matrix solution to part (a) to 15 and to 0 as appropriate. Usually they obtained two sets of solutions a = 3 with b = -3 and a = -5 with b = 5. They then discarded the second set of solutions as they had been given the condition a >0, but some candidates failed to discard the second set and lost the final A1 mark.

Question 6

In part (a) candidates could usually evaluate both f(1) and f(2) correctly and also provided a suitable conclusion. Common errors occurred where candidates incorrectly worked in degrees or failed to provide an appropriate conclusion. In some cases candidates failed to give any conclusion at all.

The work in (b) was often sound although there were a significant number of cases where candidates used negative lengths in an otherwise sound method using similar triangles. The method of similar triangles was the most common although there were other methods that were more laboured such as finding the intersection with the x-axis of the line joining f(1) and f(2) which met with varying levels of success.

Question 7

Those who understood the word singular put their determinant equal to zero and solved the subsequent equation. There were frequently sign errors leading to the solution a = - ½ and other algebraic errors leading to a = 2 instead of a = ½. In part (b) most understood the method for finding the inverse matrix, but there were a number of errors and the determinant was often given as 14 instead of 10.

Part (c) could be approached in various ways. The most popular method was to multiply the inverse matrix by the column vector with elements k – 6, and 3k + 12.

The answer obtained was the column vector with elements k and k + 3. Candidates then needed to complete their solution by concluding that the point lies on the line y = x + 3.

Another approach involved using the original matrix and multiplying it by the column vector $\begin{pmatrix} x \\ x+3 \end{pmatrix}$ and equating to $\begin{pmatrix} k-6 \\ 3k+12 \end{pmatrix}$, which leads to x = k and y = k + 3. Again a conclusion was needed.

Others used the original matrix and multiplied it by the column vector $\begin{pmatrix} x \\ y \end{pmatrix}$, again equating to $\begin{pmatrix} k-6 \\ 3k+12 \end{pmatrix}$. This leads straight to the equation y = x + 3. It was clear, however, that some candidates were unfamiliar with transformation work using matrices and did not set the transformation matrix first and follow it by a column matrix as required.

Question 8

Most candidates achieved the first four marks in part (a), but there were often slips in the simplification and many of those who first considered $f(k + 1) – f(k)$, obtaining a multiple of four stopped at that point. They did not continue a step further to make $f(k + 1)$ the subject of their formula and attempt to extract a common factor of 4 to show conclusively that $f(k + 1)$ was divisible by 4.

There were more completely correct answers in part (b), though a number of candidates were unclear that they had to multiply matrices to show this result. Those who multiplied the appropriate matrices sometimes made sign slips.

Candidates do need to learn the basic steps required for induction : true for $n = 1$, **if** (or assuming) true for $n = k$ **then** true for $n = k + 1$, therefore by induction true for all integer n. The presentation of their arguments is important in this form of proof and although a majority of candidates gave clear explanations - it was common to see candidates conclude "true for $n = 1$, k and $k + 1$ and so for all n", without a clear indication of an inductive argument.

Question 9

In general, the methods required for mathematical induction were well understood, but the specific requirements of this question were missed by many candidates. Statements were often ones that had been learned, rather than being used in the appropriate context. The conclusions were often ill-conceived, particularly when defining the values for which the proof was valid.

In part (a) some candidates validated the result for $n = 2$ rather than $n = 1$. Some candidates used $u_{k+1} = 4u_k – 9(k + 1)$ and a few wrote that $4(4^k) = 16^k$, but the most common error here was not taking the expression $4^{k+1} + 3k + 4$ any further and not formally proving that it is true for $n = k + 1$.

Part (b) was more successful than part (a), although a few candidates did not show sufficient working when multiplying out their matrices to justify being awarded full marks for their solution.

Statistics for FP1 Practice Paper Silver Level S2

Qu	Max Score	Modal score	Mean %	Mean score for students achieving grade: ALL	A*	A	B	C	D	E	U
2	9		84	7.59		8.67	7.82	7.16	6.46	4.92	3.63
3	6		90	5.38	5.90	5.78	5.47	5.33	4.98	4.77	3.79
3	9		86	7.70	8.70	8.54	7.90	7.24	6.47	5.80	4.13
4	6		93	5.60	5.94	5.87	5.80	5.70	5.61	4.82	4.13
5	8		79	6.29		7.25	6.17	5.52	4.89	4.05	2.89
6	5		76	3.81	4.86	4.52	3.95	3.49	3.00	2.39	1.74
7	8		76	6.07		7.38	5.95	5.14	4.11	3.25	1.97
8	14		73	10.26		12.36	10.11	8.62	7.12	5.48	3.42
9	10	10	69	6.85	9.57	8.79	7.20	5.79	4.13	2.96	1.63
	75		**79**	**59.55**		**69.16**	**60.37**	**53.99**	**46.77**	**38.44**	**27.33**