GCE Examinations

Decision Mathematics
Module D2

Advanced Subsidiary / Advanced Level

Paper E

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Craig Hunter, Edited by Shaun Armstrong

© *Solomon Press*

These sheets may be copied for use solely by the purchaser's institute.

1. A glazing company runs a promotion for a special type of window. As a result of this the company receives orders for 30 of these windows from business B_1, 18 from business B_2 and 22 from business B_3. The company has stocks of 20 of these windows at factory F_1, 35 at factory F_2 and 15 at factory F_3. The table below shows the profit, in pounds, that the company will make for each window it sells according to which factory supplies each business.

	B_1	B_2	B_3
F_1	20	14	17
F_2	18	19	19
F_3	15	17	23

The glazing company wishes to supply the windows so that the total profit is a maximum.

Formulate this information as a linear programming problem.

(a) State your decision variables. **(1 mark)**

(b) Write down the objective function in terms of your decision variables. **(2 marks)**

(c) Write down the constraints and state what each one represents. **(3 marks)**

© *Solomon Press*

2. *This question should be answered on the sheet provided.*

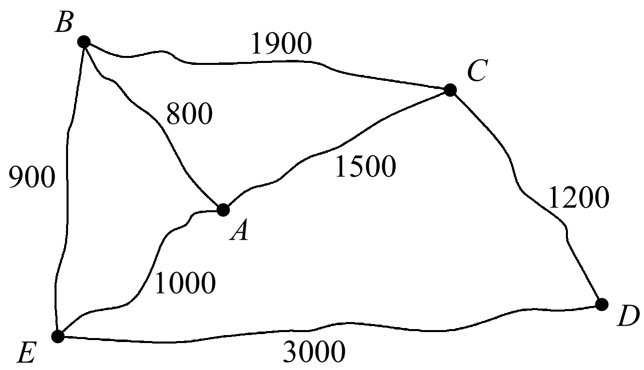


Fig. 1

Figure 1 shows a network in which the nodes represent five major rides in a theme park and the arcs represent paths between these rides. The numbers on the arcs give the length, in metres, of the paths.

(a) By inspection, add additional arcs to make a complete network showing the shortest distances between the rides.

(2 marks)

(b) Use the nearest neighbour algorithm, starting at A, and your complete network to find an upper bound to the length of a tour visiting each ride exactly once.

(3 marks)

(c) Interpret the tour found in part *(b)* in terms of the original network. **(2 marks)**

Turn over

© *Solomon Press*

3. Whilst Clive is in hospital, four of his friends decide to redecorate his lounge as a welcome-home surprise. They divide the work to be done into four jobs which must be completed in the following order:

- strip the wallpaper,
- paint the woodwork and ceiling,
- hang the new wallpaper,
- replace the fittings and tidy up.

The table below shows the time, in hours, that each of the friends is likely to take to complete each job.

	Alice	Bhavin	Carl	Dieter
Strip wallpaper	5	3	5	4
Paint	7	5	6	4
Hang wallpaper	8	4	7	6
Replace fittings	5	3	2	3

As they do not know how long Clive will be in hospital his friends wish to complete the redecoration in the shortest possible total time.

(a) Use the Hungarian method to obtain the optimal allocation of the jobs, showing the state of the table after each stage in the algorithm.

(6 marks)

(b) Hence, find the minimum time in which the friends can redecorate the lounge.

(1 mark)

© *Solomon Press*

4. *This question should be answered on the sheet provided.*

The owner of a small plane is planning a journey from her local airport, A to the airport nearest her parents, K. On the journey she will make three refuelling stops, the first at B, C or D, the second at E, F or G and the third at H, I or J.

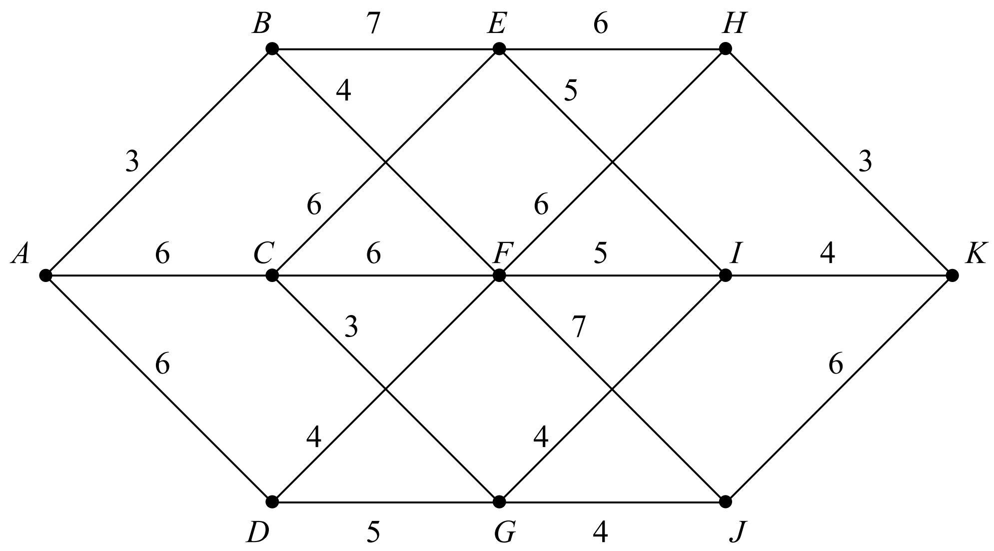


Fig. 2

Figure 2 shows all the possible flights that can be made on the journey with the number by each arc indicating the distance of each flight in hundreds of miles. As her plane does not have a large fuel tank, the owner wishes to choose a route that minimises the maximum distance of any one flight.

Find the route that she should use and state the maximum distance of any one stage on this route.

(10 marks)

Turn over

© *Solomon Press*

5. A car-hire firm has six branches in a region. Three of the branches, *A*, *B* and *C*, have spare cars, whereas the other three, *D*, *E* and *F*, require cars. The total number of cars required is equal to the number of cars available. The table below shows the cost in pounds of sending one car from each branch with spares to each branch needing more cars and the number of cars available or required by each branch.

Branches with spare cars \ Branches needing cars	*D*	*E*	*F*	Available
A	6	4	7	7
B	8	5	3	8
C	4	4	2	5
Required	5	9	6	

(a) Use the north-west corner method to obtain a possible pattern of moving cars and find its cost.

(4 marks)

The firm wishes to minimise the cost of redistributing the cars.

(b) Calculate shadow costs for the pattern found in part *(a)* and improvement indices for each unoccupied cell.

(5 marks)

(c) State, with a reason, whether or not the pattern found in part *(a)* is optimal. **(1 mark)**

© *Solomon Press*

6. *This question should be answered on the sheet provided.*

A furniture company in Leeds is considering opening outlets in six other cities.
The table below shows the distances, in miles, between all seven cities.

	Leeds	Liverpool	Manchester	Newcastle	Nottingham	Oxford	York
Leeds	–	71	40	96	71	165	28
Liverpool	71	–	31	155	92	155	93
Manchester	40	31	–	136	62	141	67
Newcastle	96	155	136	–	156	250	78
Nottingham	71	92	62	156	–	94	78
Oxford	165	155	141	250	94	–	172
York	28	93	67	78	78	172	–

(a) Starting with Leeds, obtain and draw a minimum spanning tree for this network of cities showing your method clearly. **(4 marks)**

A representative of the company is to visit each of the areas being considered. He wishes to plan a journey of minimum length starting and ending in Leeds and visiting each of the other cities in the table once.

Assuming that the network satisfies the triangle inequality,

(b) find an initial upper bound for the length of his journey, **(2 marks)**

(c) improve this upper bound to find an upper bound of less than 575 miles. **(4 marks)**

(d) By deleting Leeds, find a lower bound for his journey. **(4 marks)**

Turn over

© *Solomon Press*

7. The payoff matrix for player A in a two-person zero-sum game is shown below.

		B		
		I	II	III
A	I	$^{-}2$	3	$^{-}1$
	II	4	$^{-}5$	2

(a) Formulate this information as a linear programming problem, the solution to which will give the optimal strategy for player B.

(7 marks)

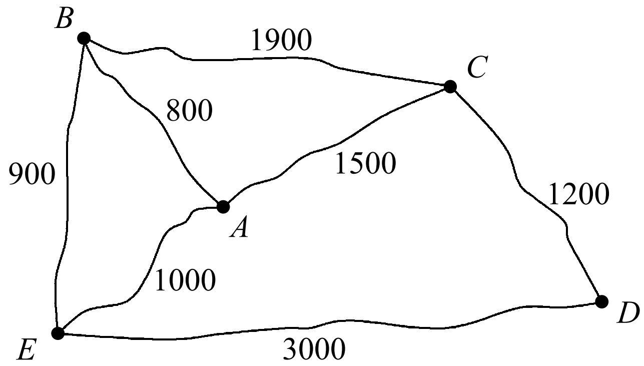
(b) By solving this linear programming problem, find the optimal strategy for player B and the value of the game.

(14 marks)

END

Please hand this sheet in for marking

(a)



………………………………………………………………………………………

………………………………………………………………………………………

………………………………………………………………………………………

(b) ………………………………………………………………………………………

………………………………………………………………………………………

………………………………………………………………………………………

………………………………………………………………………………………

(c) ………………………………………………………………………………………

………………………………………………………………………………………

………………………………………………………………………………………

………………………………………………………………………………………

© *Solomon Press*

Please hand this sheet in for marking

Stage	State	Action		
1	*H*	*HK*		
	I	*IK*		
	J	*JK*		
2	*E*	*EH* *EI*		
	F	*FH* *FI* *FJ*		
	G	*GI* *GJ*		
3	*B*	*BE* *BF*		
	C	*CE* *CF* *CG*		
	D	*DF* *DG*		
4	*A*	*AB* *AC* *AD*		

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

© *Solomon Press*

Sheet for answering question 6   *NAME* ……………………………………….

Please hand this sheet in for marking

(a)

	Leeds	Liverpool	Manchester	Newcastle	Nottingham	Oxford	York
Leeds	–	71	40	96	71	165	28
Liverpool	71	–	31	155	92	155	93
Manchester	40	31	–	136	62	141	67
Newcastle	96	155	136	–	156	250	78
Nottingham	71	92	62	156	–	94	78
Oxford	165	155	141	250	94	–	172
York	28	93	67	78	78	172	–

(b) ……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

(c)

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

Turn over

© *Solomon Press*

Sheet for answering question 6 (cont.)

(d)

	Leeds	Liverpool	Manchester	Newcastle	Nottingham	Oxford	York
Leeds	–	71	40	96	71	165	28
Liverpool	71	–	31	155	92	155	93
Manchester	40	31	–	136	62	141	67
Newcastle	96	155	136	–	156	250	78
Nottingham	71	92	62	156	–	94	78
Oxford	165	155	141	250	94	–	172
York	28	93	67	78	78	172	–

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

© *Solomon Press*