GCE Examinations

Advanced Subsidiary / Advanced Level

Decision Mathematics
Module D2

Paper E

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

Written by Craig Hunter, Edited by Shaun Armstrong

D2 Paper E – Marking Guide

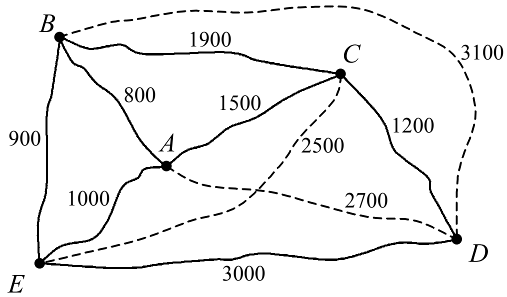
1. *(a)* x_{11} – number of windows from F_1 to B_1
x_{12} – number of windows from F_1 to B_2
x_{13} – number of windows from F_1 to B_3
x_{21} – number of windows from F_2 to B_1
x_{22} – number of windows from F_2 to B_2
x_{23} – number of windows from F_2 to B_3
x_{31} – number of windows from F_3 to B_1
x_{32} – number of windows from F_3 to B_2
x_{33} – number of windows from F_3 to B_3 B1

(b) maximise
$z = 20x_{11} + 14x_{12} + 17x_{13} + 18x_{21} + 19x_{22} + 19x_{23} + 15x_{31} + 17x_{32} + 23x_{33}$ B2

(c) $x_{11} + x_{12} + x_{13} = 20$ number of windows at F_1
$x_{21} + x_{22} + x_{23} = 35$ number of windows at F_2
$x_{31} + x_{32} + x_{33} = 15$ number of windows at F_3
$x_{11} + x_{21} + x_{31} = 30$ number of windows ordered by B_1
$x_{12} + x_{22} + x_{32} = 18$ number of windows ordered by B_2 M1 A1
$x_{13} + x_{23} + x_{33} = 22$ number of windows ordered by B_3
$x_{ij} \geq 0$ for all i, j
reference to balance B1 **(6)**

---

2. *(a)*



add AD – 2700, BD – 3100, CE – 2500 M1 A1

(b) AB (800), BE (900), EC (2500), CD (1200), DA (2700) M1 A1
tour: $ABECDA$
upper bound = 800 + 900 + 2500 + 1200 + 2700 = 8100 m A1

(c) actual tour is $ABEACDCA$ as EC and DA are not in original network M1 A1 **(7)**

---

3. *(a)*

$$\begin{array}{cccc|c} & & & & \text{row min.} \\ 5 & 3 & 5 & 4 & 3 \\ 7 & 5 & 6 & 4 & 4 \\ 8 & 4 & 7 & 6 & 4 \\ 5 & 3 & 2 & 3 & 2 \end{array}$$

reducing rows gives:

$$\begin{array}{lcccc} & 2 & 0 & 2 & 1 \\ & 3 & 1 & 2 & 0 \\ & 4 & 0 & 3 & 2 \\ & 3 & 1 & 0 & 1 \\ \hline \text{col min.} & 2 & 0 & 0 & 0 \end{array}$$

M1 A1

reducing columns gives:

$$\begin{array}{cccc} 0^* & 0 & 2 & 1 \\ 1 & 1 & 2 & 0^* \\ 2 & 0^* & 3 & 2 \\ 1 & 1 & 0^* & 1 \end{array}$$

A1

4 lines are required to cover all zeros so allocation is possible B1

strip wallpaper – Alice
paint – Dieter
hang wallpaper – Bhavin
replace fittings – Carl M1 A1

(b) 5 + 4 + 4 + 2 = 15 hours A1 **(7)**

4.

Stage	State	Action	Destination	Value	
1	*H*	*HK*	*K*	3*	
	I	*IK*	*K*	4*	A1
	J	*JK*	*K*	6*	
2	*E*	*EH* *EI*	*H* *I*	max(6, 3) = 6 max(5, 4) = 5*	
	F	*FH* *FI* *FJ*	*H* *I* *J*	max(6, 3) = 6 max(5, 4) = 5* max(7, 6) = 7	
	G	*GI* *GJ*	*I* *J*	max(4, 4) = 4* max(4, 6) = 6	M1 A2
3	*B*	*BE* *BF*	*E* *F*	max(7, 5) = 7 max(4, 5) = 5*	
	C	*CE* *CF* *CG*	*E* *F* *G*	max(6, 5) = 6 max(6, 5) = 6 max(3, 4) = 4*	
	D	*DF* *DG*	*F* *G*	max(4, 5) = 5* max(5, 4) = 5*	M1 A1
4	*A*	*AB* *AC* *AD*	*B* *C* *D*	max(3, 5) = 5* max(6, 4) = 6 max(6, 5) = 6	A1

giving route *ABFIK* M1 A1
maximum stage length = 500 miles A1 **(10)**

---

5. *(a)*

	D	E	F	Available
A	5	2		7
B		7	1	8
C			5	5
Required	5	9	6	

M1 A1

cost = $(5 \times 6) + (2 \times 4) + (7 \times 5) + (1 \times 3) + (5 \times 2) = £86$ M1 A1

(b) taking $R_1 = 0$,
$R_1 + K_1 = 6 \;\therefore\; K_1 = 6$
$R_1 + K_2 = 4 \;\therefore\; K_2 = 4$
$R_2 + K_2 = 5 \;\therefore\; R_2 = 1$
$R_2 + K_3 = 3 \;\therefore\; K_3 = 2$ M1 A2
$R_3 + K_3 = 2 \;\therefore\; R_3 = 0$

	$K_1 = 6$	$K_2 = 4$	$K_3 = 2$
$R_1 = 0$	0	0	7
$R_2 = 1$	8	0	0
$R_3 = 0$	4	4	0

improvement indices, $I_{ij} = C_{ij} - R_i - K_j$

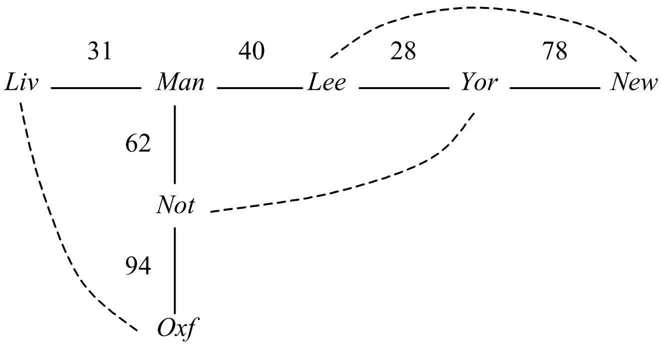
$\therefore\; I_{13} = 7 - 0 - 2 = 5$
$I_{21} = 8 - 1 - 6 = 1$
$I_{31} = 4 - 0 - 6 = {}^{-}2$ M1 A1
$I_{32} = 4 - 0 - 4 = 0$

(c) pattern not optimal as there is a negative improvement index B1 **(10)**

6. *(a)*

order:	1	4	3	6	5	7	2
	Lee	*Liv*	*Man*	*New*	*Not*	*Oxf*	*Yor*
Lee	–	71	40	96	71	165	28
Liv	71	–	$\boxed{31}$	155	92	155	93
Man	$\boxed{40}$	31	–	136	62	141	67
New	96	155	136	–	156	250	$\boxed{78}$
Not	71	92	$\boxed{62}$	156	–	94	78
Oxf	165	155	141	250	$\boxed{94}$	–	172
Yor	$\boxed{28}$	93	67	78	78	172	–

M1 A2



A1

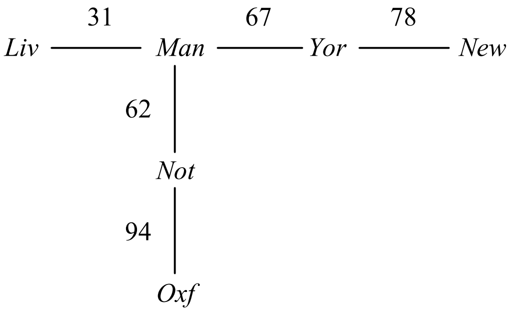
(b) upper bound = 2 × weight of MST
$= 2 \times (31 + 40 + 28 + 78 + 62 + 94) = 2 \times 333 = 666$ miles M1 A1

(c) use *Liv* – *Oxf* saving $31 + 62 + 94 - 155 = 32$
use *Not* – *Yor* saving $62 + 40 + 28 - 78 = 52$
use *Lee* – *New* saving $28 + 78 - 96 = 10$ M1 A2
new upper bound = $666 - 32 - 52 - 10 = 572$ miles A1

(d) e.g. starting at *Liv*

order:		1	2	5	3	6	4
	Lee	*Liv*	*Man*	*New*	*Not*	*Oxf*	*Yor*
Lee	–	71	40	96	71	165	28
Liv	71	–	31	155	92	155	93
Man	40	$\boxed{31}$	–	136	62	141	67
New	96	155	136	–	156	250	$\boxed{78}$
Not	71	92	$\boxed{62}$	156	–	94	78
Oxf	165	155	141	250	$\boxed{94}$	–	172
Yor	28	93	$\boxed{67}$	78	78	172	–

M1



A1

lower bound = weight of MST + two edges of least weight from *Lee*
$= (31 + 67 + 78 + 62 + 94) + 28 + 40 = 400$ miles M1 A1 **(14)**

7. *(a)* adding 6 to all entries to make them positive gives: M1

		B		
		I	II	III
A	I	4	9	5
	II	10	1	8

let B play strategies I, II and III with proportions p_1, p_2 and p_3 M1
let value of altered game be v

let $x_1 = \frac{p_1}{v}$, $x_2 = \frac{p_2}{v}$, $x_3 = \frac{p_3}{v}$, $P = \frac{1}{v}$ M1

from A I, $4p_1 + 9p_2 + 5p_3 \le v$
from A II, $10p_1 + p_2 + 8p_3 \le v$ M1 A1
also, $p_1 + p_2 + p_3 = 1$

dividing by v problem becomes

maximise $P = x_1 + x_2 + x_3$
subject to $4x_1 + 9x_2 + 5x_3 \le 1$
$10x_1 + x_2 + 8x_3 \le 1$ M1 A1
$x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0$

(b) using slack variables r and s gives
$4x_1 + 9x_2 + 5x_3 + r = 1$
$10x_1 + x_2 + 8x_3 + s = 1$ M1

tableau 1:

Basic Var.	x_1	x_2	x_3	r	s	Value
r	4	9	5	1	0	1
s	$\boxed{10}$	1	8	0	1	1
P	$^-1$	$^-1$	$^-1$	0	0	0

A1

taking $\boxed{10}$ as pivot M1

tableau 2:

Basic Var.	x_1	x_2	x_3	r	s	Value
r	0	$\boxed{8\frac{3}{5}}$	$1\frac{4}{5}$	1	$-\frac{2}{5}$	$\frac{3}{5}$
x_1	1	$\frac{1}{10}$	$\frac{4}{5}$	0	$\frac{1}{10}$	$\frac{1}{10}$
P	0	$-\frac{9}{10}$	$-\frac{1}{5}$	0	$\frac{1}{10}$	$\frac{1}{10}$

M1 A2

taking $\boxed{8\frac{3}{5}}$ as pivot M1

tableau 3:

Basic Var.	x_1	x_2	x_3	r	s	Value
x_2	0	1	$\frac{9}{43}$	$\frac{5}{43}$	$-\frac{2}{43}$	$\frac{3}{43}$
x_1	1	0	$\boxed{\frac{67}{86}}$	$-\frac{1}{86}$	$\frac{9}{86}$	$\frac{4}{43}$
P	0	0	$-\frac{1}{86}$	$\frac{9}{86}$	$\frac{5}{86}$	$\frac{7}{43}$

M1 A1

taking $\boxed{\frac{67}{86}}$ as pivot

tableau 4:

Basic Var.	x_1	x_2	x_3	r	s	Value
x_2	$-\frac{18}{67}$	1	0	$\frac{8}{67}$	$-\frac{5}{67}$	$\frac{3}{67}$
x_3	$1\frac{19}{67}$	0	1	$-\frac{1}{67}$	$\frac{9}{67}$	$\frac{8}{67}$
P	$\frac{1}{67}$	0	0	$\frac{7}{67}$	$\frac{4}{67}$	$\frac{11}{67}$

A1

tableau is optimal

$x_1 = 0,\ x_2 = \frac{3}{67},\ x_3 = \frac{8}{67},\ P = \frac{1}{v} = \frac{11}{67} \quad \therefore v = \frac{67}{11}$ M1

giving $p_1 = 0,\ p_2 = \frac{67}{11} \times \frac{3}{67} = \frac{3}{11},\ p_3 = \frac{67}{11} \times \frac{8}{67} = \frac{8}{11}$

$\therefore$ *B* should not play I, should play II $\frac{3}{11}$ of time and III $\frac{8}{11}$ of time M1 A1

value of original game = $\frac{67}{11} - 6 = \frac{1}{11}$ A1 **(21)**

Total **(75)**

Performance Record – D2 Paper E

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	transport., formulate lin. prog.	nearest neighbour	allocation	dynamic prog., minimax	transport., n-w corner, improv. indices	TSP, shortcuts	game, formulate lin. prog. and solve	
Marks	6	7	7	10	10	14	21	75
Student								