GCE Examinations

Advanced Subsidiary / Advanced Level

Decision Mathematics
Module D2

Paper D

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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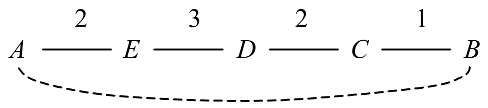
D2 Paper D – Marking Guide

1. *(a)*

order:	1	5	4	3	2
	A	*B*	*C*	*D*	*E*
A	–	4	7	8	2
B	4	–	1	5	6
C	7	1	–	2	7
D	8	5	2	–	3
E	2	6	7	3	–

M1



A1

upper bound = 2 × weight of MST

$= 2 \times (2 + 3 + 2 + 1) = 2 \times 8 = 16$ miles — M1 A1

(b) use *AB* saving $2 + 3 + 2 + 1 - 4 = 4$ — M1

new upper bound = 16 – 4 = 12 miles — A1 **(6)**

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2. *(a)* adding 5 to all entries to make them positive gives — M1

		B		
		I	II	III
A	I	11	1	4
	II	3	10	8
	III	10	6	2

new value of game $v = V + 5$ — A1

(b) let *B* play strategies I, II and III with proportions p_1, p_2 and p_3 — M1

let $x_1 = \frac{p_1}{v}$, $x_2 = \frac{p_2}{v}$, $x_3 = \frac{p_3}{v}$ — A1

(c) $p_1 + p_2 + p_3 = 1$ — M1

dividing by v gives $x_1 + x_2 + x_3 = \frac{1}{v}$

we wish to minimise v ∴ maximise $\frac{1}{v}$

objective function is maximise $P = x_1 + x_2 + x_3$ — A1

(d) from *A* I, $11p_1 + p_2 + 4p_3 \le v$

from *A* II, $3p_1 + 10p_2 + 8p_3 \le v$ — M1

from *A* III, $10p_1 + 6p_2 + 2p_3 \le v$

dividing by v gives the constraints

$$11x_1 + x_2 + 4x_3 \le 1$$
$$3x_1 + 10x_2 + 8x_3 \le 1$$
$$10x_1 + 6x_2 + 2x_3 \le 1$$ — A1 **(8)**

also $x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0$

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3. *(a)*

order:	1	6	2	3	5	4	7
	A	*B*	*C*	*D*	*E*	*F*	*G*
A	–	83	57	68	103	91	[120]
B	83	–	78	63	[41]	82	52
C	[57]	78	–	37	59	63	74
D	68	63	[37]	–	60	52	62
E	103	41	59	60	–	[48]	51
F	91	82	63	[52]	48	–	77
G	120	[52]	74	62	51	77	–

M1 A1

tour: *ACDFEBGA* A1

upper bound = 57 + 37 + 52 + 48 + 41 + 52 + 120 = 407 miles A1

(b) e.g. starting at *B*

order:		1	6	5	2	3	4
	A	*B*	*C*	*D*	*E*	*F*	*G*
A	–	83	57	68	103	91	120
B	83	–	78	63	41	82	52
C	57	78	–	[37]	59	63	74
D	68	63	37	–	60	[52]	62
E	103	[41]	59	60	–	48	51
F	91	82	63	52	[48]	–	77
G	120	52	74	62	[51]	77	–

M1 A1

lower bound = weight of MST + two edges of least weight from *A*

= (41 + 48 + 51 + 52 + 37) + 57 + 68 = 354 miles M1 A1

(c) $354 \le d \le 407$ B1 **(9)**

4.

Stage	State	Action	Destination	Value	
1	*I*	*IL*	*L*	5*	
	J	*JL*	*L*	6*	A1
	K	*KL*	*L*	10*	
2	*F*	*FI* *FJ* *FK*	*I* *J* *K*	min(5, 5) = 5* min(2, 6) = 2 min(2, 10) = 2	
	G	*GI* *GJ* *GK*	*I* *J* *K*	min(8, 5) = 5 min(9, 6) = 6* min(3, 10) = 3	
	H	*HI* *HJ* *HK*	*I* *J* *K*	min(10, 5) = 5 min(2, 6) = 2 min(9, 10) = 9*	M1 A2
3	*B*	*BF* *BG* *BH*	*F* *G* *H*	min(8, 5) = 5 min(11, 6) = 6* min(4, 9) = 4	
	C	*CF* *CH*	*F* *H*	min(5, 5) = 5 min(10.5, 9) = 9*	
	D	*DF* *DH*	*F* *H*	min(9, 5) = 5 min(6, 9) = 6*	
	E	*EF* *EG* *EH*	*F* *G* *H*	min(12, 5) = 5 min(7, 6) = 6 min(15, 9) = 9*	M1 A1
4	*A*	*AB* *AC* *AD* *AE*	*B* *C* *D* *E*	min(1, 6) = 1 min(4.5, 9) = 4.5 min(13, 6) = 6 min(10, 9) = 9*	A1

giving route *AEHKL* M1 A1
shortest stage is 9 miles A1 **(10)**

5. need to add dummy column giving — M1

	19	69	168	0
	22	64	157	0
	20	72	166	0
	23	66	171	0
col min.	19	64	157	0

reducing rows will make no difference — B1

reducing columns gives:

0	5	11	0
3	0	0	0
1	8	9	0
4	2	14	0

(N.B. a different choice of lines will lead to the same final assignment) — M1 A1

3 lines required to cover all zeros, apply algorithm — B1

0	5	11	1
3	0	0	1
0	7	8	0
3	1	13	0

M1 A1

3 lines required to cover all zeros, apply algorithm

0*	4	10	1
4	0	0*	2
0	6	7	0*
3	0*	12	0

A1

4 lines required to cover all zeros so allocation is possible — B1

stage 1 is run by Alex
stage 2 is run by Suraj
stage 3 is run by Darren — M1 A1 **(11)**
Leroy does not take part

6. *(a)*

		Y_1	Y_2	row minimum
X	X_1	$^-2$	4	$^-2$
	X_2	6	$^-1$	$^-1$
column maximum		6	4	

(Column header Y spans Y_1 and Y_2.) M1 A1

max (row min) = $^-1$ min (col max) = 4
max (row min) ≠ min (col max) ∴ no saddle point B1

(b) (i) let X play strategies X_1 and X_2 with proportions p and $(1 - p)$

expected payoff to X against each of Y's strategies:

Y_1 $^-2p + 6(1 - p) = 6 - 8p$
Y_2 $4p - (1 - p) = 5p - 1$ M1 A1

for optimal strategy $6 - 8p = 5p - 1$
∴ $13p = 7,\ p = \frac{7}{13}$

∴ X should play X_1 $\frac{7}{13}$ of time and X_2 $\frac{6}{13}$ of time M1 A1

(ii) let Y play strategies Y_1 and Y_2 with proportions q and $(1 - q)$

expected loss to Y against each of X's strategies:

X_1 $^-2q + 4(1 - q) = 4 - 6q$
X_2 $6q - (1 - q) = 7q - 1$ M1 A1

for optimal strategy $4 - 6q = 7q - 1$
∴ $13q = 5,\ q = \frac{5}{13}$

∴ Y should play Y_1 $\frac{5}{13}$ of time and Y_2 $\frac{8}{13}$ of time M1 A1

(c) value of game $= 6 - (8 \times \frac{7}{13}) = 1\frac{9}{13}$ M1 A1 **(13)**

7. *(a)*

	D	E	F	Available
A	20			20
B	10	5		15
C			25	25
Required	30	5	25	

M1 A1

no. of rows + no. of cols – 1 = 3 + 3 – 1 = 5
in this solution only 4 cells are occupied, less than 5 $\therefore$ degenerate — B1

(b) placing 0 in (3, 2) as it has lowest cost of unoccupied cells

taking $R_1 = 0$, $R_1 + K_1 = 13 \therefore K_1 = 13$ $\quad R_2 + K_1 = 10 \therefore R_2 = {}^-3$
$R_2 + K_2 = 9 \therefore K_2 = 12$ $\quad R_3 + K_2 = 6 \therefore R_3 = {}^-6$ — M1 A2
$R_3 + K_3 = 8 \therefore K_3 = 14$

	$K_1 = 13$	$K_2 = 12$	$K_3 = 14$
$R_1 = 0$	(0)	11	14
$R_2 = {}^-3$	(0)	(0)	12
$R_3 = {}^-6$	15	(0)	(0)

improvement indices, $I_{ij} = C_{ij} - R_i - K_j$

$\therefore$ $I_{12} = 11 - 0 - 12 = {}^-1$
$I_{13} = 14 - 0 - 14 = 0$
$I_{23} = 12 - ({}^-3) - 14 = 1$ — M1 A1
$I_{31} = 15 - ({}^-6) - 13 = 8$

pattern not optimal as there is a negative improvement index — B1

applying algorithm

	D	E	F
A	$20 - \theta$	θ	
B	$10 + \theta$	$5 - \theta$	
C			25

let $\theta = 5$, giving

	D	E	F
A	15	5	
B	15		
C			25

M1 A1

this solution is also degenerate
place 0 in (3, 2) again

taking $R_1 = 0$, $R_1 + K_1 = 13 \therefore K_1 = 13$ $\quad R_1 + K_2 = 11 \therefore K_2 = 11$
$R_2 + K_1 = 10 \therefore R_2 = {}^-3$ $\quad R_3 + K_2 = 6 \therefore R_3 = {}^-5$ — M1 A1
$R_3 + K_3 = 8 \therefore K_3 = 13$

	$K_1 = 13$	$K_2 = 11$	$K_3 = 13$
$R_1 = 0$	(0)	(0)	14
$R_2 = {}^-3$	(0)	9	12
$R_3 = {}^-5$	15	(0)	(0)

$\therefore$ $I_{13} = 14 - 0 - 13 = 1$
$I_{22} = 9 - ({}^-3) - 11 = 1$
$I_{23} = 12 - ({}^-3) - 13 = 2$ — M1 A1
$I_{31} = 15 - ({}^-5) - 13 = 7$

all improvement indices are non-negative $\therefore$ pattern is optimal — B1

15 units from A to D, 5 units from A to E,
15 units from B to D, 25 units from C to F — A1

total cost = (15 × 13) + (5 × 11) + (15 × 10) + (25 × 8) = £600 — A1 **(18)**

Total **(75)**

Performance Record – D2 Paper D

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	TSP, shortcuts	game, formulate lin. prog.	TSP, nearest neighbour	dynamic prog., maximin	allocation, dummy	game	transport., n-w corner, stepping-stone, degeneracy	
Marks	6	8	9	10	11	13	18	75
Student								