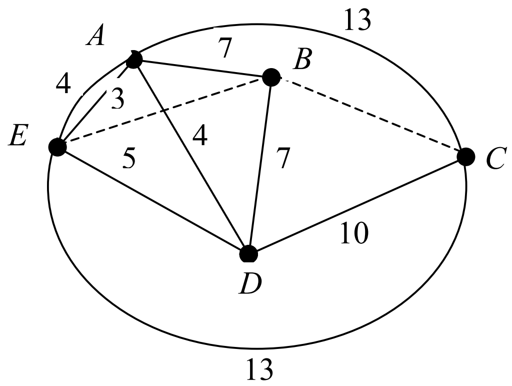
Question number	Scheme	Marks
1. (*a*)	x_{11} no. of coaches from *A* to *D*	
	x_{12} no. of coaches from *A* to *E*	
	x_{13} no. of coaches from *A* to *F*	
	x_{21} no. of coaches from *B* to *D*	
	x_{22} no. of coaches from *B* to *E*	
	x_{23} no. of coaches from *B* to *F*	
	x_{31} no. of coaches from *C* to *D*	
	x_{32} no. of coaches from *C* to *E*	
	x_{33} no. of coaches from *C* to *F*	B1 (1)
(*b*)	Minimise $z = 40x_{11} + 70x_{12} + 25x_{13}$	
	$+ 20x_{21} + 40x_{22} + 10x_{23}$	
	$+ 35x_{31} + 85x_{32} + 15x_{33}$	B1 (1)
(*c*)	Depot *A* $x_{11} + x_{12} + x_{13} = 8$ (no. of coaches at *A*)	
	Depot *B* $x_{21} + x_{22} + x_{23} = 5$ (no. of coaches at *B*)	
	Depot *C* $x_{31} + x_{32} + x_{33} = 7$ (no. of coaches at *C*)	M1 A1
	Depot *D* $x_{11} + x_{21} + x_{31} = 4$ (no. required at *D*)	
	Depot *E* $x_{21} + x_{22} + x_{32} = 10$ (no. required at *E*)	
	Depot *F* $x_{31} + x_{32} + x_{33} = 6$ (no. required at *F*)	M1 A1
	Reference to number of coaches at *A*, *B* and *C*	B1 (5)
	= number of coaches at *D*, *E* and *F*	

Question number		Scheme	Marks	
2.	(*a*)			
		BC: 17	M1	
		EB: 10	A1	(2)
	(*b*)	*AE*(3), *ED*(5), *DB*(7), *BC*(17)	M1 A1	
		Complete with edge *CA*(13)		
		Total length 45 km	A1	(3)
	(*c*)	Tour in original is *A E D B D C A*		
		Since *BC* is not in original network and shortest distance is *BD* plus *DC*	M1 A1	(2)
				(7 marks)

Question number	Scheme	Marks
3.	Suppose A chooses I with probability p	
	A chooses II with probability $(1-p)$	
	Expected gain if B chooses I $\quad 4p-5(1-p)$	
	II $\quad -2p+6(1-p)$	M1 A1
	Optimal value when	
	$4p-5(1-p)=-2p+6(1-p)$	
	$p=\frac{11}{17},\ 1-p=\frac{6}{17}$	
	Play I, $\frac{11}{17}$ of time and II, $\frac{6}{17}$ of time	M1 A1
	Suppose B chooses I with probability q	
	B chooses II with probability $(1-q)$	
	Expected loss if A chooses I $\quad 4q-2(1-q)$	
	II $\quad -5q+6(1-q)$	M1 A1
	Optimal value when	
	$4q-2(1-q)=-5q+6(1-q)$	
	$q=\frac{8}{17},\ 1-q=\frac{9}{17}$	M1 A1
	Play I, $\frac{8}{17}$ of time and II, $\frac{9}{17}$ of time	
	Value of game $=9p-5(=4p-5(1-p))=\frac{14}{17}$ gain to player A	M1 A1 (10)
	[or $6-11q=-5q+6(1-q)$] $=\frac{14}{17}$ loss to Player B]	
		(10 marks)

Question number	Scheme	Marks

4. (*a*)

	W_1	W_2	W_3	Available
F_1	2	2		4
F_2		3		3
F_3		4	4	8
Require	2	9	4	

M1 A1 A1

Cost $2 \times 7 + 2 \times 8 + 3 \times 2 + 4 \times 6 + 4 \times 3$

$=14 + 16 + 6 + 24 + 12=72$ — M1 A1 (5)

(*b*) For occupied cells $R_i + K_j = C_{ij}$ gives

$(1,1)\, R_1 + K_1 = 7; (1,2)\, R_1 + K_2 = 8; (2,2)\, R_2 + K_2 = 2$

$(3,2)\, R_3 + K_2 = 6; (3,3)\, R_3 + K_3 = 3$ — M1 A1

Taking $R_1 = 0$ we obtain $K_1 = 7,\ K_2 = 8,\ R_2 = -6,\ R_3 = -2,\ K_3 = 5$

	$K_1 = 7$	$K_2 = 8$	$K_3 = 5$
$R_1 = 0$	0	0	6
$R_2 = -6$	9	0	4
$R_3 = -2$	5	0	0

M1 A1

Improvement indices $I_{ij} = C_{ij} - R_i - K_j$

$I_{13} = 6 - 5 - 0 = 1$

$I_{21} = 9 - 7 - (-6) = 8$

$I_{23} = 4 - 5 - (-6) = 5$

$I_{31} = 5 - 7 - (-2) = 0$ — M1 A1 (6)

(*c*) No negative improvement indices and so given solution is optimal and gives minimum cost. If there were a negative I_{ij} then using this route would reduce cost. — M1 (1)

(12 marks)

Question number		Stage	State	Action	Cost	Value	Marks
5.	(a)	2	0	*B*	2	2	
				C	3	3 ←	
			1	*A*	2	2	
				B	3	3	
				C	6	6 ←	
			2	*A*	1	1	
				B	2	2 ←	M1 A1 A1
		1	0	*B*	2	2 + 3 = 5	
				C	3	3 + 6 = 9 ←	M1 A1
			1	*A*	1	1 + 3 = 4	
				B	3	3 + 6 = 9 ←	
				C	6	6 + 2 = 8	A1
			2	*A*	5	5 + 6 =11 ←	
				B	5	5 + 2 = 7	
							A1
		0	0	*A*	4	4 + 9 = 13	
				B	3	3 + 9 = 12	
				C	5	5 + 11=16 ←	M1 A1 (9)
	(b)	Hence maximum profit is 16					B1
		Tracing back through calculations the optimal strategy is *C A C*					M1 A1 (3)
							(12 marks)

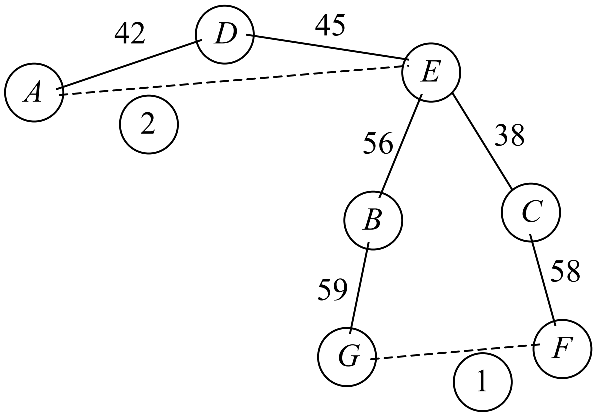
Question number	Scheme	Marks
6. (a)	Row minimum	
	$\begin{array}{cccc\vdots c} 17 & 24 & 19 & 18 & 17 \\ 12 & 23 & 16 & 15 & 12 \\ 16 & 24 & 21 & 18 & 16 \\ 12 & 24 & 18 & 14 & 12 \end{array}$	
	Reducing rows gives: $\begin{array}{cccc} 0 & 7 & 2 & 1 \\ 0 & 11 & 4 & 3 \\ 0 & 8 & 5 & 2 \\ 0 & 12 & 6 & 2 \end{array}$	
	Column minimum $\begin{array}{cccc} 0 & 7 & 2 & 1 \end{array}$	M1 A1
	Reducing columns gives: $\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 4 & 2 & 2 \\ 0 & 1 & 3 & 1 \\ 0 & 5 & 4 & 1 \end{array}$	M1 A1
	No assignment possible as zeroes can all be covered by 2 lines (2<4)	B1
	Minimum uncovered element is 1	
	Applying algorithm gives:	
	$\begin{array}{cccc} 1 & 0 & 0^* & 0 \\ 0^* & 3 & 1 & 1 \\ 0 & 0^* & 2 & 0 \\ 0 & 4 & 3 & 0^* \end{array}$	M1 A1, A1
	Now requires 4 lines to cover all zeroes so assignment now possible	B1
	(1, 3) - only zero in column 3	
	(3, 2) - row 1 already used and now only zero in $C2$	
	(4, 4) - only remaining possibility in $C4$	
	(2, 1) - must then be used	
	I – C, II – A, III – B, IV – D	M1 A1 (11)
(b)	Cost of this assignment	
	19 + 12 + 24 + 14 = 69 minutes	M1 A1 (2)
		(13 marks)

Question number	Scheme	Marks

7. (a)

	A 1	*B* 5	*C* 4	*D* 2	*E* 3	*F* 6	*G* 7	Marks
	-	103	89	42	54	143	153	
B	103	-	60	98	56	99	59	
C	89	60	-	65	38	58	77	
D	42	98	65	-	45	111	139	
E	54	56	38	45	-	95	100	
F	143	99	58	111	95	-	75	
G	153	59	77	139	100	75	-	M1 A1 A1

Vertices added in order *ADECBFG*



M1 A1 (5)

(b) Upper bound = 2 (weight of M.S.T.)

= 2(42 + 45 + 38 + 58 + 56 + 59)

= 2(298) = 596 — M1 A1 (2)

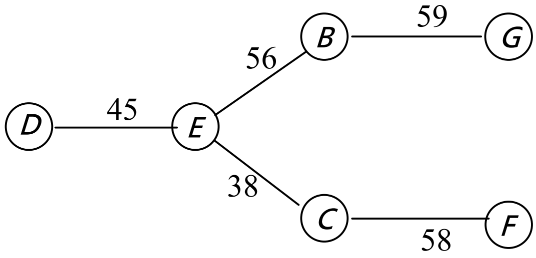
(c) Short cut 1 replaces *FCEBG* by *FG* saving (58 + 38 + 56 + 59) – 75 = 136

Now upper bound is 460 — M1 A1

Short cut 2 replaces *EDA* by *EA* saving (45 + 42) – 54 = 33

Now upper bound is 427 — A1 (3)

continued over…

7.	(d)	If *A* is removed then M.S.T. of remaining network is  	M1 A1
		Lower bound is obtained by adding weights of edges *AD*(42) and *AE*(54) (edges of least weight at *A*)	
		So lower bounds is 256 + 42 + 54 = 352	M1 A1 (4)
			(14 marks)