FOR EDEXCEL

GCE Examinations
Advanced Subsidiary

Core Mathematics C4

Paper K

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and / or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has seven questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working may gain no credit.

Written by Shaun Armstrong

1.

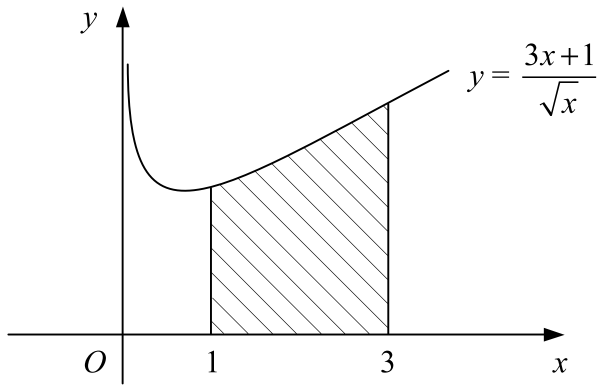

Figure 1

Figure 1 shows the curve with equation $y = \dfrac{3x+1}{\sqrt{x}}$, $x > 0$.

The shaded region is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 3$.

Find the volume of the solid formed when the shaded region is rotated through 2π radians about the x-axis, giving your answer in the form $\pi(a + \ln b)$, where a and b are integers. **(6)**

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2. *(a)* Expand $(1 - 3x)^{-2}$, $|x| < \frac{1}{3}$, in ascending powers of x up to and including the term in x^3, simplifying each coefficient. **(4)**

(b) Hence, or otherwise, show that for small x,

$$\left(\frac{2-x}{1-3x}\right)^2 \approx 4 + 20x + 85x^2 + 330x^3.$$ **(3)**

3.
$$\mathrm{f}(x) = \frac{7+3x+2x^2}{(1-2x)(1+x)^2}, \quad |x| > \tfrac{1}{2}.$$

(a) Express $\mathrm{f}(x)$ in partial fractions. **(4)**

(b) Show that

$$\int_1^2 \mathrm{f}(x)\ \mathrm{d}x = p - \ln q,$$

where p is rational and q is an integer. **(7)**

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4. Relative to a fixed origin, two lines have the equations

$$\mathbf{r} = \begin{pmatrix} 7 \\ 0 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ 4 \\ -2 \end{pmatrix}$$

and
$$\mathbf{r} = \begin{pmatrix} a \\ 6 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 14 \\ 2 \end{pmatrix},$$

where a is a constant and λ and μ are scalar parameters.

Given that the two lines intersect,

(a) find the position vector of their point of intersection, **(5)**

(b) find the value of a. **(2)**

Given also that θ is the acute angle between the lines,

(c) find the value of $\cos\theta$ in the form $k\sqrt{5}$ where k is rational. **(4)**

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5. A curve has the equation

$$x^2 - 4xy + 2y^2 = 1.$$

(a) Find an expression for $\frac{dy}{dx}$ in its simplest form in terms of x and y. **(5)**

(b) Show that the tangent to the curve at the point P (1, 2) has the equation

$$3x - 2y + 1 = 0.$$ **(3)**

The tangent to the curve at the point Q is parallel to the tangent at P.

(c) Find the coordinates of Q. **(4)**

Turn over

6. The rate of increase in the number of bacteria in a culture, N, at time t hours is proportional to N.

(a) Write down a differential equation connecting N and t. **(1)**

Given that initially there are N_0 bacteria present in a culture,

(b) Show that $N = N_0 e^{kt}$, where k is a positive constant. **(6)**

Given also that the number of bacteria present doubles every six hours,

(c) find the value of k, **(3)**

(d) find how long it takes for the number of bacteria to increase by a factor of ten, giving your answer to the nearest minute. **(3)**

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7. A curve has parametric equations

$$x = \sec\theta + \tan\theta, \quad y = \operatorname{cosec}\theta + \cot\theta, \quad 0 < \theta < \tfrac{\pi}{2}.$$

(a) Show that $x + \frac{1}{x} = 2\sec\theta$. **(5)**

Given that $y + \frac{1}{y} = 2\operatorname{cosec}\theta$,

(b) find a cartesian equation for the curve. **(3)**

(c) Show that $\frac{dx}{d\theta} = \frac{1}{2}(x^2 + 1)$. **(3)**

(d) Find an expression for $\frac{dy}{dx}$ in terms of x and y. **(4)**

END