FOR EDEXCEL

GCE Examinations
Advanced Subsidiary

Core Mathematics C4

Paper H

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and / or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has eight questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working may gain no credit.

Written by Shaun Armstrong

1. *(a)* Expand $(1 + 4x)^{\frac{3}{2}}$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient. **(4)**

(b) State the set of values of x for which your expansion is valid. **(1)**

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2. Use the substitution $u = 1 + \sin x$ to find the value of

$$\int_0^{\frac{\pi}{2}} \cos x\,(1 + \sin x)^3 \;\mathrm{d}x.$$ **(6)**

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3. *(a)* Express $\dfrac{x+11}{(x+4)(x-3)}$ as a sum of partial fractions. **(3)**

(b) Evaluate

$$\int_0^2 \frac{x+11}{(x+4)(x-3)}\;\mathrm{d}x,$$

giving your answer in the form $\ln k$, where k is an exact simplified fraction. **(5)**

4.

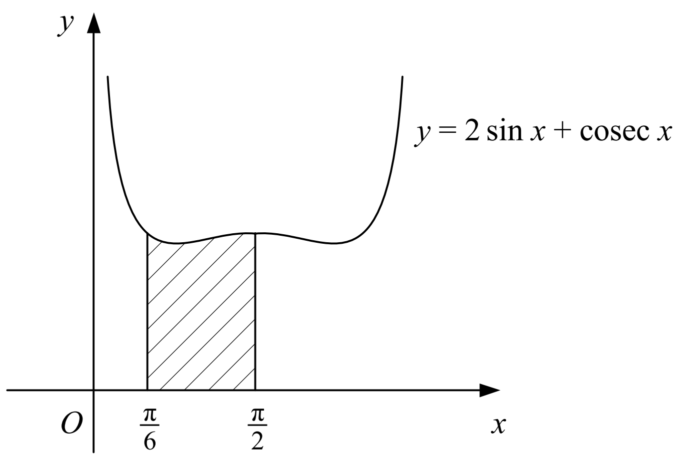

Figure 1

Figure 1 shows the curve with equation $y = 2\sin x + \operatorname{cosec} x$, $0 < x < \pi$.

The shaded region bounded by the curve, the x-axis and the lines $x = \frac{\pi}{6}$ and $x = \frac{\pi}{2}$ is rotated through 360° about the x-axis.

Show that the volume of the solid formed is $\frac{1}{2}\pi(4\pi + 3\sqrt{3})$. **(8)**

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5. A curve has the equation

$$x^2 - 3xy - y^2 = 12.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of x and y. **(5)**

(b) Find an equation for the tangent to the curve at the point (2, –2). **(3)**

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6. Relative to a fixed origin, O, the points A and B have position vectors $\begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 3 \\ -6 \end{pmatrix}$ respectively.

Find, in exact, simplified form,

(a) the cosine of $\angle AOB$, **(4)**

(b) the area of triangle OAB, **(4)**

(c) the shortest distance from A to the line OB. **(2)**

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7. A curve has parametric equations

$$x = t(t - 1), \quad y = \frac{4t}{1-t}, \quad t \neq 1.$$

(a) Find $\frac{dy}{dx}$ in terms of t. **(4)**

The point P on the curve has parameter $t = -1$.

(b) Show that the tangent to the curve at P has the equation

$$x + 3y + 4 = 0.$$ **(3)**

The tangent to the curve at P meets the curve again at the point Q.

(c) Find the coordinates of Q. **(7)**

Turn over

8. An entomologist is studying the population of insects in a colony.

Initially there are 300 insects in the colony and in a model, the entomologist assumes that the population, P, at time t weeks satisfies the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP,$$

where k is a constant.

(a) Find an expression for P in terms of k and t. **(5)**

Given that after one week there are 360 insects in the colony,

(b) find the value of k to 3 significant figures. **(2)**

Given also that after two and three weeks there are 440 and 600 insects respectively,

(c) comment on suitability of the model. **(2)**

An alternative model assumes that

$$\frac{\mathrm{d}P}{\mathrm{d}t} = P(0.4 - 0.25\cos 0.5t).$$

(d) Using the initial data, $P = 300$ when $t = 0$, solve this differential equation. **(4)**

(e) Compare the suitability of the two models. **(3)**

END