FOR EDEXCEL

GCE Examinations
Advanced Subsidiary

Core Mathematics C4

Paper G

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and / or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has eight questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working may gain no credit.

Written by Shaun Armstrong

1. A curve has the equation

$$x^2 + 2xy^2 + y = 4.$$

Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. **(6)**

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2. Use integration by parts to find

$$\int x^2 \mathrm{e}^{-x} \ \mathrm{d}x.$$ **(7)**

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3. The first four terms in the series expansion of $(1 + ax)^n$ in ascending powers of x are

$$1 - 4x + 24x^2 + kx^3,$$

where a, n and k are constants and $|ax| < 1$.

(a) Find the values of a and n. **(6)**

(b) Show that $k = -160$. **(2)**

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4. *(a)* Use the trapezium rule with two intervals of equal width to find an estimate for the value of the integral

$$\int_0^3 \mathrm{e}^{\cos x} \ \mathrm{d}x,$$

giving your answer to 3 significant figures. **(5)**

(b) Use the trapezium rule with four intervals of equal width to find another estimate for the value of the integral to 3 significant figures. **(2)**

(c) Given that the true value of the integral lies between the estimates made in parts *(a)* and *(b)*, comment on the shape of the curve $y = \mathrm{e}^{\cos x}$ in the interval $0 \leq x \leq 3$ and explain your answer. **(2)**

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5. A straight road passes through villages at the points A and B with position vectors $(9\mathbf{i} - 8\mathbf{j} + 2\mathbf{k})$ and $(4\mathbf{j} + \mathbf{k})$ respectively, relative to a fixed origin.

The road ends at a junction at the point C with another straight road which lies along the line with equation

$$\mathbf{r} = (2\mathbf{i} + 16\mathbf{j} - \mathbf{k}) + \mu(-5\mathbf{i} + 3\mathbf{j}),$$

where μ is a scalar parameter.

(a) Find the position vector of C. **(5)**

Given that 1 unit on each coordinate axis represents 200 metres,

(b) find the distance, in kilometres, from the village at A to the junction at C. **(4)**

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6. A small town had a population of 9000 in the year 2001.

In a model, it is assumed that the population of the town, P, at time t years after 2001 satisfies the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = 0.05P\mathrm{e}^{-0.05t}.$$

(a) Show that, according to the model, the population of the town in 2011 will be 13 300 to 3 significant figures. **(7)**

(b) Find the value which the population of the town will approach in the long term, according to the model. **(3)**

Turn over

7.

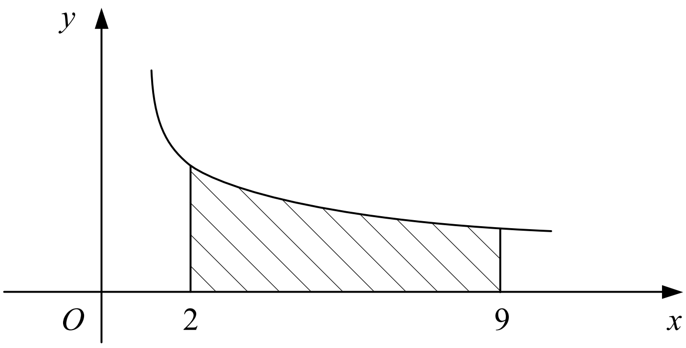

Figure 1

Figure 1 shows the curve with parametric equations

$$x = t^3 + 1, \quad y = \frac{2}{t}, \quad t > 0.$$

The shaded region is bounded by the curve, the x-axis and the lines $x = 2$ and $x = 9$.

(a) Find the area of the shaded region. **(5)**

(b) Show that the volume of the solid formed when the shaded region is rotated through 2π radians about the x-axis is 12π. **(3)**

(c) Find a cartesian equation for the curve in the form $y = \mathrm{f}(x)$. **(3)**

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8. *(a)* Show that the substitution $u = \sin x$ transforms the integral

$$\int \frac{6}{\cos x(2 - \sin x)}\, \mathrm{d}x$$

into the integral

$$\int \frac{6}{(1 - u^2)(2 - u)}\, \mathrm{d}u.$$ **(4)**

(b) Express $\dfrac{6}{(1 - u^2)(2 - u)}$ in partial fractions. **(4)**

(c) Hence, evaluate

$$\int_0^{\frac{\pi}{6}} \frac{6}{\cos x(2 - \sin x)}\, \mathrm{d}x,$$

giving your answer in the form $a \ln 2 + b \ln 3$, where a and b are integers. **(7)**

END