FOR EDEXCEL

GCE Examinations
Advanced Subsidiary

Core Mathematics C4

Paper E

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and / or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has eight questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working may gain no credit.

Written by Shaun Armstrong

1. Find

$$\int \cot^2 2x \ \mathrm{d}x. \qquad \textbf{(4)}$$

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2. A curve has the equation

$$4 \cos x + 2 \sin y = 3.$$

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2 \sin x \sec y$. **(5)**

(b) Find an equation for the tangent to the curve at the point $(\frac{\pi}{3}, \frac{\pi}{6})$, giving your answer in the form $ax + by = c$, where a and b are integers. **(3)**

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3. *(a)* Express $\dfrac{2+20x}{1+2x-8x^2}$ as a sum of partial fractions. **(4)**

(b) Hence find the series expansion of $\dfrac{2+20x}{1+2x-8x^2}$, $|x| < \frac{1}{4}$, in ascending powers of x up to and including the term in x^3, simplifying each coefficient. **(5)**

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4. The line l_1 passes through the points P and Q with position vectors $(-\mathbf{i} - 8\mathbf{j} + 3\mathbf{k})$ and $(2\mathbf{i} - 9\mathbf{j} + \mathbf{k})$ respectively, relative to a fixed origin.

(a) Find a vector equation for l_1. **(2)**

The line l_2 has the equation

$$\mathbf{r} = (6\mathbf{i} + a\mathbf{j} + b\mathbf{k}) + \mu(\mathbf{i} + 4\mathbf{j} - \mathbf{k})$$

and also passes through the point Q.

(b) Find the values of the constants a and b. **(3)**

(c) Find, in degrees to 1 decimal place, the acute angle between lines l_1 and l_2. **(4)**

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5. At time $t = 0$, a tank of height 2 metres is completely filled with water. Water then leaks from a hole in the side of the tank such that the depth of water in the tank, y metres, after t hours satisfies the differential equation

$$\frac{dy}{dt} = -ke^{-0.2t},$$

where k is a positive constant,

(a) Find an expression for y in terms of k and t. **(4)**

Given that two hours after being filled the depth of water in the tank is 1.6 metres,

(b) find the value of k to 4 significant figures. **(3)**

Given also that the hole in the tank is h cm above the base of the tank,

(c) show that $h = 79$ to 2 significant figures. **(3)**

6.

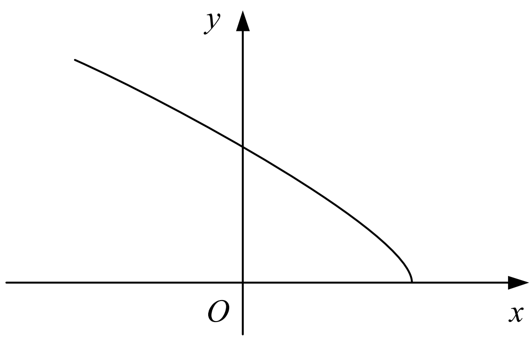

Figure 1

Figure 1 shows the curve with parametric equations

$$x = 2 - t^2, \quad y = t(t + 1), \quad t \geq 0.$$

(a) Find the coordinates of the points where the curve meets the coordinate axes. **(4)**

(b) Find the exact area of the region bounded by the curve and the coordinate axes. **(6)**

Turn over

7. *(a)* Prove that

$$\frac{\mathrm{d}}{\mathrm{d}x}(a^x) = a^x \ln a. \qquad \textbf{(3)}$$

A curve has the equation $y = 4^x - 2^{x-1} + 1$.

(b) Show that the tangent to the curve at the point where it crosses the y-axis has the equation

$$3x \ln 2 - 2y + 3 = 0. \qquad \textbf{(5)}$$

(c) Find the exact coordinates of the stationary point of the curve. **(4)**

8.

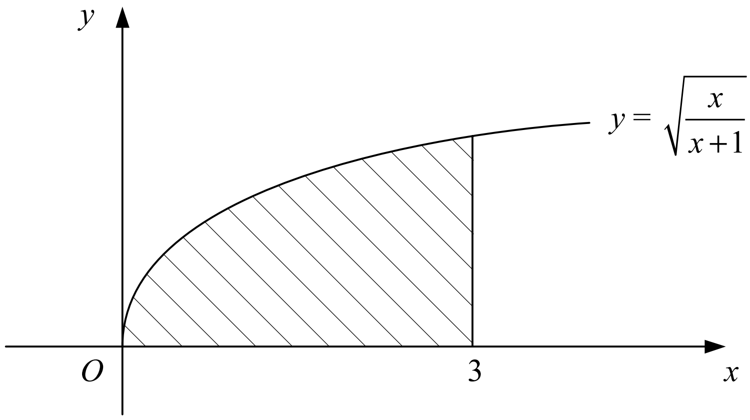

Figure 2

Figure 2 shows the curve with equation $y = \sqrt{\frac{x}{x+1}}$.

The shaded region is bounded by the curve, the x-axis and the line $x = 3$.

(a) *(i)* Use the trapezium rule with three strips to find an estimate for the area of the shaded region.

(ii) Use the trapezium rule with six strips to find an improved estimate for the area of the shaded region. **(7)**

The shaded region is rotated through 2π radians about the x-axis.

(b) Show that the volume of the solid formed is $\pi(3 - \ln 4)$. **(6)**

END