GCE Examinations
Advanced Subsidiary

Core Mathematics C4

Paper B
Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and / or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has eight questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working may gain no credit.

Written by Shaun Armstrong

1. Use integration by parts to find

$$\int x^2 \sin x \, dx.$$ **(6)**

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2. Given that $y = -2$ when $x = 1$, solve the differential equation

$$\frac{dy}{dx} = y^2 \sqrt{x},$$

giving your answer in the form $y = f(x)$. **(7)**

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3. A curve has the equation

$$4x^2 - 2xy - y^2 + 11 = 0.$$

Find an equation for the normal to the curve at the point with coordinates $(-1, -3)$. **(8)**

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4. *(a)* Expand $(1 + ax)^{-3}$, $|ax| < 1$, in ascending powers of x up to and including the term in x^3. Give each coefficient as simply as possible in terms of the constant a. **(3)**

Given that the coefficient of x^2 in the expansion of $\frac{6-x}{(1+ax)^3}$, $|ax| < 1$, is 3,

(b) find the two possible values of a. **(4)**

Given also that $a < 0$,

(c) show that the coefficient of x^3 in the expansion of $\frac{6-x}{(1+ax)^3}$ is $\frac{14}{9}$. **(2)**

5.

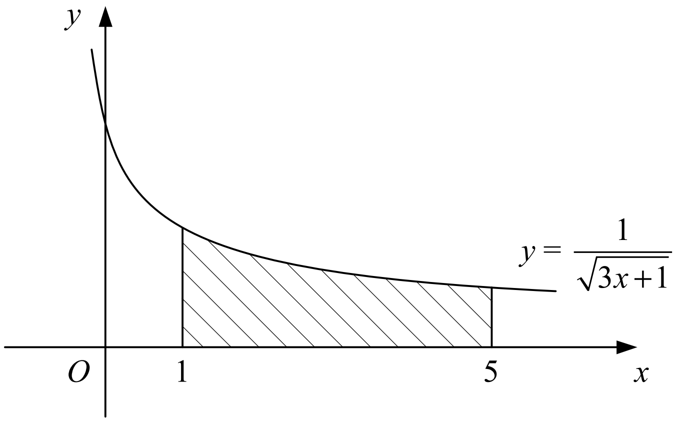

Figure 1

Figure 1 shows the curve with equation $y = \dfrac{1}{\sqrt{3x+1}}$.

The shaded region is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 5$.

(a) Find the area of the shaded region. **(4)**

The shaded region is rotated completely about the x-axis.

(b) Find the volume of the solid formed, giving your answer in the form $k\pi \ln 2$, where k is a simplified fraction. **(5)**

6.

$$\text{f}(x) = \frac{15-17x}{(2+x)(1-3x)^2}, \quad x \neq -2, \quad x \neq \tfrac{1}{3}.$$

(a) Find the values of the constants A, B and C such that

$$\text{f}(x) = \frac{A}{2+x} + \frac{B}{1-3x} + \frac{C}{(1-3x)^2}.$$ **(4)**

(b) Find the value of

$$\int_{-1}^{0} \text{f}(x)\ \text{d}x,$$

giving your answer in the form $p + \ln q$, where p and q are integers. **(7)**

Turn over

7.

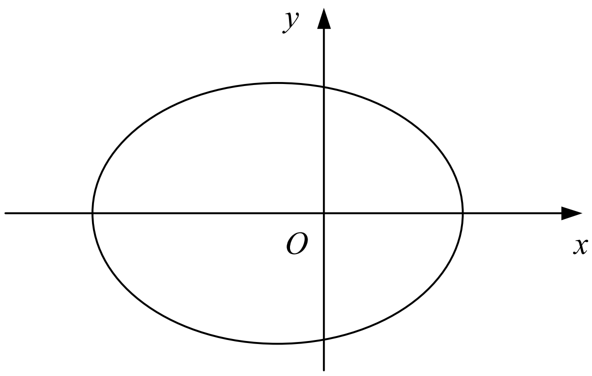

Figure 2

Figure 2 shows the curve with parametric equations

$$x = -1 + 4\cos\theta,\quad y = 2\sqrt{2}\sin\theta,\quad 0 \leq \theta < 2\pi.$$

The point P on the curve has coordinates $(1, \sqrt{6})$.

(a) Find the value of θ at P. **(2)**

(b) Show that the normal to the curve at P passes through the origin. **(7)**

(c) Find a cartesian equation for the curve. **(3)**

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8. The line l_1 passes through the points A and B with position vectors $(-3\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$ and $(7\mathbf{i} - \mathbf{j} + 12\mathbf{k})$ respectively, relative to a fixed origin.

(a) Find a vector equation for l_1. **(2)**

The line l_2 has the equation

$$\mathbf{r} = (5\mathbf{j} - 7\mathbf{k}) + \mu(\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}).$$

The point C lies on l_2 and is such that AC is perpendicular to BC.

(b) Show that one possible position vector for C is $(\mathbf{i} + 3\mathbf{j})$ and find the other. **(8)**

Assuming that C has position vector $(\mathbf{i} + 3\mathbf{j})$,

(c) find the area of triangle ABC, giving your answer in the form $k\sqrt{5}$. **(3)**

END