GCE Examinations

Advanced Subsidiary

Core Mathematics C4

Paper A

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and / or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has seven questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working may gain no credit.

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Written by Shaun Armstrong

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1. A curve has the equation

$$x^2(2 + y) - y^2 = 0.$$

Find an expression for $\dfrac{dy}{dx}$ in terms of x and y. **(6)**

2.
$$f(x) = \frac{3}{\sqrt{1-x}}, \quad |x| < 1.$$

(a) Show that $f(\frac{1}{10}) = \sqrt{10}$. **(2)**

(b) Expand $f(x)$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient. **(3)**

(c) Use your expansion to find an approximate value for $\sqrt{10}$, giving your answer to 8 significant figures. **(1)**

(d) Find, to 1 significant figure, the percentage error in your answer to part *(c)*. **(2)**

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3. Relative to a fixed origin, O, the line l has the equation

$$\mathbf{r} = (\mathbf{i} + p\mathbf{j} - 5\mathbf{k}) + \lambda(3\mathbf{i} - \mathbf{j} + q\mathbf{k}),$$

where p and q are constants and λ is a scalar parameter.

Given that the point A with coordinates $(-5, 9, -9)$ lies on l,

(a) find the values of p and q, **(3)**

(b) show that the point B with coordinates $(25, -1, 11)$ also lies on l. **(2)**

The point C lies on l and is such that OC is perpendicular to l.

(c) Find the coordinates of C. **(4)**

(d) Find the ratio $AC : CB$ **(2)**

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4. During a chemical reaction, a compound is being made from two other substances. At time t hours after the start of the reaction, x g of the compound has been produced. Assuming that $x = 0$ initially, and that

$$\frac{dx}{dt} = 2(x - 6)(x - 3),$$

(a) show that it takes approximately 7 minutes to produce 2 g of the compound. **(10)**

(b) Explain why it is not possible to produce 3 g of the compound. **(2)**

5.
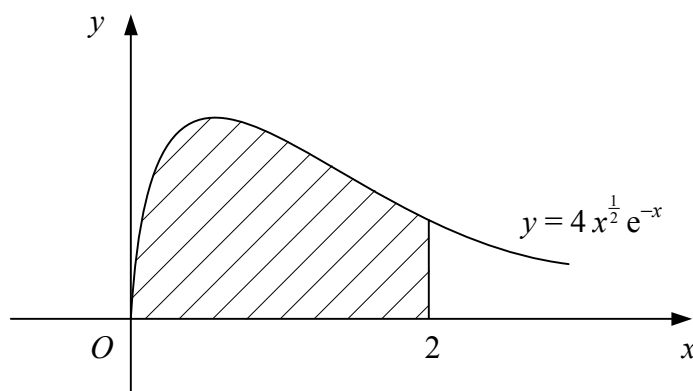
Figure 1

Figure 1 shows the curve with equation $y = 4\,x^{\frac{1}{2}}\,e^{-x}$.

The shaded region is bounded by the curve, the x-axis and the line $x = 2$.

(a) Use the trapezium rule with four intervals of equal width to estimate the area of the shaded region. **(5)**

The shaded region is rotated through 2π radians about the x-axis.

(b) Find, in terms of π and e, the exact volume of the solid formed. **(7)**

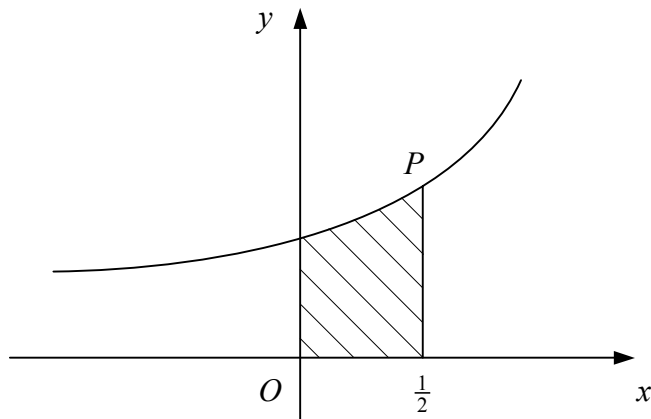**6.** *(a)* Find

$$\int 2 \sin 3x \sin 2x \; dx.$$ **(4)**

(b) Use the substitution $u^2 = x + 1$ to evaluate

$$\int_0^3 \frac{x^2}{\sqrt{x+1}} \; dx.$$ **(8)**

7.

Figure 2

Figure 2 shows the curve with parametric equations

$$x = \cos 2t, \quad y = \operatorname{cosec} t, \quad 0 < t < \tfrac{\pi}{2}.$$

The point P on the curve has x-coordinate $\tfrac{1}{2}$.

(a) Find the value of the parameter t at P. **(2)**

(b) Show that the tangent to the curve at P has the equation

$$y = 2x + 1.$$ **(5)**

The shaded region is bounded by the curve, the coordinate axes and the line $x = \tfrac{1}{2}$.

(c) Show that the area of the shaded region is given by

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} k \cos t \ dt,$$

where k is a positive integer to be found. **(4)**

(d) Hence find the exact area of the shaded region. **(3)**

END